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Some classes of shrinkage estimators in the morgenstern type bivariate exponential distribution using ranked set sampling

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Abstract

This article proposes a class of shrinkage estimators of Morgenstern type bivariate exponential distribution (MTBED) based on concomitants of order statistic in ranked set sampling (RSS). The class of estimators for the parameter is motivated by the work of Jani (1991). The proposed class of shrinkage estimators has smaller mean square error (MSE) than the Chacko and Thomas (2008) estimators and minimum mean squared error (MMSE) estimators for wider range of the parameter. Numerical computations indicate that certain of these estimators substantially improve the usual and minimum mean squared error (MMSE) estimators for value of the parameter near the prior estimate, especially for small sample sizes.

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Keywords: Ranked set sampling, Morgenstern type bivariate exponential distribution, Concomitants of order statistic, Minimum mean square error estimator, Shrinkage estimator.

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1. Introduction

The concept of Ranked set sampling (RSS) was first introduced by McIntyre (1952) as a process of improving the precision of the sample mean as an estimator of the population mean. Ranked set sampling as described in McIntyre (1952) is applicable whenever ranking of a set of sampling units can be done easily by a judgement method (for a detailed discussion on the theory and applications of ranked set sampling see, Chen et al., (2004)). Ranking by judgement method is not recommendable if the judgement method is too crude and is not powerful for ranking by discriminating the units of a moderately large sample. In certain situations, one may prefer exact measurement of some easily measurable variable associated with the study variable rather than ranking the units by a crude judgement method. Suppose the variable of interest say Y, is difficult or much expensive to measure, but an auxiliary variable X correlated with Y is readily measurable and can be ordered exactly. In this case as an alternative to McIntyre (1952) method of RSS, Stokes (1977) used an auxiliary variable for the ranking of the sampling units. If $X_{r(r)}$ is the observation measured on the auxiliary variable X from the unit chosen from the rth set then we write $Y_{r[r]}$ to denote the corresponding measurement made on the study variable Y on this unit, then $Y_{r[r]}, r = 1, 2, ..., n$, form the ranked set sample. Clearly $Y_{r[r]}$ is the concomitant of the rth order statistic arising from the rth sample.

Chacko and Thomas (2008) assumed a Morgenstern type bivariate exponential distribution (MTBED) corresponding to bivariate random variable (X, Y), where X denote the auxiliary variable and Y denote the study variable with probability density function (pdf) as

$$f_{XY}(x,y) = \frac{e^{\frac{-x}{\theta_1}} e^{\frac{-y}{\theta_2}}}{\theta_1\theta_2}[1 + \alpha(1 - 2e^{\frac{-x}{\theta_1}})(1 - 2e^{\frac{-y}{\theta_2}})]; \tag{1.1}$$

$$x > 0, y > 0, \theta_1 > 0, \theta_2 > 0, -1 \leq \alpha \leq 1.$$

Stokes (1995) has considered the estimation of parameters of location-scale family of distributions using RSS. Lam et al. (1994, 1995) have obtained the best linear unbiased estimators (BLUEs) of location and scale parameters of exponential distribution and logistic distribution. The Fisher information contained in RSS have been discussed by Chen (2000) and Chen and Bai (2000). Stokes (1980) has considered the method of estimation of correlation coefficient of bivariate normal distribution using RSS. Modarres and Zheng (2004) have considered the problem of estimation of dependence parameter using RSS. Robust estimate of correlation coefficient for bivariate normal distribution have been developed by Zheng and Modarres (2006). Stokes (1977) has suggested the ranked set sample mean as an estimator for the mean of the study variate Y, when an auxiliary variable X is used for ranking the sample units, under the assumption that (X, Y) follows a bivariate normal distribution. Barnett and Moore (1997) have improved the estimator of Stokes (1977) by deriving the BLUE of the mean of the study variate Y, based on ranked set sample obtained on the study variate Y . Al-Saleh and Al-Kadiri (2000) have extended first the usual concept of RSS to double stage ranked set sampling (DSRSS) with an objective of increasing the precision of certain estimators of the population when compared with those obtained based on usual RSS or using random sampling. Al-Saleh and Al-Omari (2002) have further extended DSRSS to multistage ranked set sampling (MSRSS) and shown that there is increase in the precision of estimators obtained based on MSRSS when compared with those based on usual RSS and DSRSS. Al-Saleh (2004) has considered the steady-state RSS.

The remaining plan of the paper is given as follows: In section 2 we have discussed a brief discussion on Chacko and Thomas (2008) estimators in MTBED using RSS. Section 3 dealt with some minimum mean squared error (MMSE) estimators on the lines of Searls (1964), Singh et al. (1973) and Searls and Intarapanich (1990) along with their properties. In section 4 we have proposed some shrinkage estimators of the parameter θ_2 in MTBED on the lines of Jani (1991) and Kourouklis (1994). We have also obtained their biases and mean squared errors (MSEs) and shown theoretically that the shrinkage estimators are superior estimate of θ_2 as compared to Chacko and Thomas (2008) estimators and MMSE estimators. In section 5 we have computed the relative efficiencies of different estimators numerically to evaluate their performance. Section 6 concludes the paper with final comments.

2. Chacko and Thomas (2008) estimators based on ranked set sampling (RSS) in Morgenstern type bivariate exponential distribution (MTBED)

Let (X, Y) be a bivariate random variable which follows a MTBED with pdf defined by (1.1). Let $X_{r(r)}$ be the observation measured on the auxiliary variate X in the rth unit of the RSS and let $Y_{r[r]}$ be the measurement made on the Y variate of the same unit,$r = 1, 2, ..., n$. Then clearly $Y_{r[r]}$ is distributed as the concomitant of rth order statistic of a random sample of size n arising from (1.1). By using the expressions for means and variances of concomitants of order statistics arising from MTBED obtained by Scaria and Nair (1999), the mean and variance of $Y_{r[r]}$ for $-1 \leq \alpha \leq 1$ are given as

$$E[Y_{r[r]}] = \theta_2\left[1 - \frac{\alpha}{2}\left(\frac{n-2r+1}{n+1}\right)\right] = \theta_2\xi_r(say). \tag{2.1}$$

$$Var[Y_{r[r]}] = \theta_2^2\left[1 - \frac{\alpha}{2}\left(\frac{n-2r+1}{n+1}\right) - \frac{\alpha^2}{4}\left(\frac{n-2r+1}{n+1}\right)^2\right] = \theta_2^2\delta_r(say). \tag{2.2}$$

Chacko and Thomas (2008) shows ranked set sample mean as

$$t_1 = \theta_2^* = \frac{1}{n}\sum_{r=1}^{n} Y_{r[r]}, \tag{2.3}$$

is an unbiased estimator of θ_2 and its variance is given by

$$Var(t_1) = \frac{\theta_2^2}{n}\left[1 - \frac{\alpha^2}{4n}\sum_{r=1}^{n}(\frac{n-2r+1}{n+1})^2\right] = \theta_2^2 V_1, \tag{2.4}$$

where $V_1 = \frac{1}{n}\left[1 - \frac{\alpha^2}{4n}\sum_{r=1}^{n}(\frac{n-2r+1}{n+1})^2\right]$.

Chacko and Thomas (2008) further provided a better estimator of θ_2 than that of θ_2^* by deriving the BLUE $\hat{\theta}_2$ of θ_2 provided the parameter α is known as

$$t_2 = \hat{\theta}_2 = \frac{\sum_{r=1}^{n}(\frac{\xi_r}{\delta_r})Y_{r[r]}}{\sum_{r=1}^{n}(\frac{\xi_r^2}{\delta_r})}, \tag{2.5}$$

and

$$Var(t_2) = \frac{\theta_2^2}{\sum_{r=1}^{n}(\frac{\xi_r^2}{\delta_r})} = \theta_2^2 V_2, \tag{2.6}$$

where $V_2 = \frac{1}{\sum_{r=1}^{n}(\frac{\xi_r^2}{\delta_r})}$.
Chacko and Thomas (2008) further obtained BLUE based on single stage unbalanced RSS as

$$t_3 = \hat{\theta}_2^{n(1)} = \frac{1}{n\xi_n}\sum_{i=1}^{n} Y_{[n]i}, \tag{2.7}$$

and

$$Var(t_3) = \frac{\theta_2^2 \delta_n}{n[1+\frac{\alpha}{2}](\xi_n)^2} = \theta_2^2 V_3, \tag{2.8}$$

where $V_3 = \frac{\delta_n}{n(\xi_n)^2}$.
Chacko and Thomas (2008) also shows BLUE based on single stage unbalanced steady-state RSS as

$$t_4 = \hat{\theta}_2^{n(\infty)} = \frac{1}{n[1+\frac{\alpha}{2}]}\sum_{i=1}^{n} Y_{n[i]}^{\infty}, \tag{2.9}$$

and

$$Var(t_4) = \frac{\theta_2^2[1+\frac{\alpha}{2}-\frac{\alpha^2}{4}]}{n[1+\frac{\alpha}{2}]^2} = \theta_2^2 V_4, \tag{2.10}$$

where $V_4 = \frac{\left[1+\frac{\alpha}{2}-\frac{\alpha^2}{4}\right]}{n\left[1+\frac{\alpha}{2}\right]^2}$.

3. Minimum mean squared error (MMSE) estimators of the parameter θ_2

The MMSE estimator of the parameter θ_2 based on $t_i's, i = 1, 2, 3, 4$ are

$$T_{im} = \frac{t_i}{(1+V_i)}, \tag{3.1}$$

in the class of estimators $T_i = A_i t_i$, where $A_i's$ are suitably chosen constants such that the MSE of $T_i's$ are minimum.
The biases and MSEs of $T_{im}'s$ are respectively given by

$$B(T_{im}) = -\theta_2\left(\frac{V_i}{(1+V_i)}\right), \tag{3.2}$$

$$MSE(T_{im}) = \theta_2^2\left(\frac{V_i}{(1+V_i)}\right). \tag{3.3}$$

From (2.4), (2.6), (2.8), (2.10) and (3.3) we have that

$$Var(t_i) - MSE(T_{im}) = \frac{\theta_2^2 V_i^2}{(1+V_i)} > 0, i = 1, 2, 3, 4, . \tag{3.4}$$

which shows that $T_{im}'s, i = 1, 2, 3, 4$ are always superior to the Chacko and Thomas (2008) corresponding estimators $t_i's, i = 1, 2, 3, 4$.

4. Suggested class of estimators using θ_{20} as a prior information about θ_2

Inserting $t_i's, i = 1, 2, 3, 4$ in place of sample mean $\bar{X}$ based on simple random sampling (SRS) in Jani's (1991) class of estimators, we define a class of shrinkage estimators of the parameter θ_2

$$T_{i(p)} = \theta_{20} + k_{(p)}(t_i - \theta_{20}), i = 1, 2, 3, 4, \tag{4.1}$$

which is based on ranked set sampling in MTBED, where $k_{(p)} = \frac{\Gamma(n-p)}{n^p \Gamma(n-2p)}, p$ being a non-zero real number.
The biases and MSEs of $T_{i(p)}{}'s$ are respectively given by

$$B(T_{i(p)}) = \theta_2 \phi (1 - k_{(p)}) \tag{4.2}$$

$$MSE(T_{i(p)}) = \theta_2^2 [k_{(p)}^2 (\phi^2 + V_i) - 2\phi^2 k_{(p)} + \phi^2], \tag{4.3}$$

where $\phi = (\frac{\theta_{20}}{\theta_2} - 1) = (\lambda - 1)$ with $\lambda = (\frac{\theta_{20}}{\theta_2})$.
We now state the following theorems.
Theorem 1 The proposed estimator $T_{i(p)}{}'s, i = 1, 2, 3, 4$ are better than the corresponding unbiased estimators $t_i{}'s, i = 1, 2, 3, 4$ if

$$k_{(p)} < 1, \frac{\theta_{20}}{1 + \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}} < \theta_2 < \frac{\theta_{20}}{1 - \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}}. \tag{4.4}$$

Proof
From (2.4),(2.6),(2.8),(2.10) and (4.3) we have that
$MSE(T_{i(p)}) - Var(t_i) = \theta_2^2 (1 - k_{(p)})[\phi^2 (1 - k_{(p)}) - V_i (1 + k_{(p)})] < 0,$
if
$1 - k_{(p)} > 0, \phi^2 < \frac{(1+k_p)V_i}{(1-k_p)},$
or $k_{(p)} < 1, -\sqrt{\frac{(1+k_p)V_i}{(1-k_p)}} < \phi < \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}},$
or

$$k_{(p)} < 1, \left(1 - \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}\right) < \lambda = \left(\frac{\theta_{20}}{\theta_2}\right) < (1 + \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}\Bigg), \tag{4.5}$$

or $k_{(p)} < 1, \theta_2 \left(1 - \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}\right) < \theta_{20} < \theta_2 \left(1 + \sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}\right),$
or $k_{(p)} < 1, \frac{\theta_{20}}{1+\sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}} < \theta_2 < \frac{\theta_{20}}{1-\sqrt{\frac{(1+k_p)V_i}{(1-k_p)}}}.$
Theorem 2 The proposed estimator $T_{i(p)}{}'s, i = 1, 2, 3, 4$ are better than the corresponding MMSE estimators $T_{im}{}'s, i = 1, 2, 3, 4$ if

$$\frac{\theta_{20}}{1 + \sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}} < \theta_2 < \frac{\theta_{20}}{1 - \sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}}. \tag{4.6}$$

Proof
From (3.3) and (4.3) we have that
$MSE(T_{im}) - MSE(T_{i(p)}) = \theta_2^2 (1 + V_i)^{-1} [V_i \big(1 - k_{(p)}^2 (1 + V_i)\big)$
$- \phi^2 (1 + V_i)(1 - k_{(p)})^2] > 0,$
if
$\phi^2 < \frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2},$
or $-\sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}} < \phi < \sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}},$

or

$$(4.7)\quad \left(1-\sqrt{\frac{V_i\big(1-k_{(p)}^2(1+V_i)\big)}{(1+V_i)(1-k_p)^2}}\right)<\lambda<\left(1+\sqrt{\frac{V_i\big(1-k_{(p)}^2(1+V_i)\big)}{(1+V_i)(1-k_p)^2}}\right),$$

or $\theta_2\left(1-\sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}\right)<\theta_{20}<\theta_2\left(1+\sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}\right)$,

or $k_{(p)}<1,\ \frac{\theta_{20}}{\left(1+\sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}\right)}<\theta_2<\frac{\theta_{20}}{\left(1-\sqrt{\frac{V_i\left(1-k_{(p)}^2(1+V_i)\right)}{(1+V_i)(1-k_p)^2}}\right)}$.

It can be easily seen that the proposed shrinkage estimators $T_{i(p)}{}'s, i=1,2,3,4$ are better than the corresponding usual estimators $t_i{}'s, i=1,2,3,4$ and corresponding MMSE estimators $T_{im}{}'s, i=1,2,3,4$ for a wider range of θ_2 . The member of the class of estimators $T_{i(p)}{}'s, i=1,2,3,4$ have smaller MSE than $t_i{}'s, i=1,2,3,4$ for all (n,α) and for θ_2 in the neighborhood of θ_{20} . Largest range of dominance of λ is obtained when $p=-1$ with the resulting estimators $T_{i(-1)}=\theta_{20}+k_{(-1)}(t_i-\theta_{20})$ (see Table 3). Thus $T_{i(-1)}=\theta_{20}+k_{(-1)}(t_i-\theta_{20})$ are better than $t_i{}'s$ no matter how much θ_{20} underestimates θ_2 . Roughly speaking, $p's$ with small absolute values give wider neighborhoods of dominance of $T_{i(p)}{}'s$ over $t_i{}'s$ (see Tables 5-6).

Remark: If we have a situation with α unknown, we introduce an estimator (moment type) for α as follows. For MTBED the correlation coefficient between the two variables is given by $\rho=\frac{\alpha}{4}$. If r is the sample correlation coefficient between $X_{i(i)}$ and $Y_{i[i]}, i=1,2,...,n$ then the moment type estimator for α is obtained by equating with the population correlation coefficient ρ and is obtained as [see, Chacko and Thomas (2008)]:

$$\hat{\alpha}=\begin{cases}-1 & \text{if } r<(-1/4)\\ 4r & \text{if } (-1/4)\le r\le(1/4).\\ 1 & \text{if } r>(1/4)\end{cases}$$

5. Relative efficiency

As we have seen on computer screen that the MMSE estimator T_{4m} has the smallest MSE among the estimators $T_{im}{}'s, i=1,2,3,4$, therefore we have made the comparison of the proposed shrinkage estimators with that of T_{4m}. For this purpose we have computed the relative efficiencies of various suggested shrinkage estimators to the MMSE estimator T_{4m} by using following formulae:

$e_1=RE(T_{1(p)},T_{4m})=\frac{V_4}{(1+V_4)[k_{(p)}^2(\phi^2+V_1)-2\phi^2k_{(p)}+\phi^2]}$;

$e_2=RE(T_{2(p)},T_{4m})=\frac{V_4}{(1+V_4)[k_{(p)}^2(\phi^2+V_2)-2\phi^2k_{(p)}+\phi^2]}$;

$e_3=RE(T_{3(p)},T_{4m})=\frac{V_4}{(1+V_4)[k_{(p)}^2(\phi^2+V_3)-2\phi^2k_{(p)}+\phi^2]}$;

$e_4=RE(T_{4(p)},T_{4m})=\frac{V_4}{(1+V_4)[k_{(p)}^2(\phi^2+V_4)-2\phi^2k_{(p)}+\phi^2]}$.

The values of $e_i{}'s, i=1,2,3,4$ for $n=5(5)20, p=\pm1,\pm2, \alpha=0.25(0.25)1.00$ and different values of λ are shown in Table 1.

It is observed from Table 1 that for fixed $(n,\alpha,|p|)$, the values of $e_i{}'s, i=1,2,3,4$ increase as λ increases up to 1, while it decreases if λ goes beyond 1. When the value of λ is 'unity' (i.e. the guessed value θ_{20} coincides with the true value θ_2), the higher gain in efficiency is seen which is expected too. Also higher gain in efficiency is obtained when sample size n is small. In general the higher gain in efficiency are observed by using $T_{4(p)}$ over T_{4m} for all values of $(n,\alpha,|p|,\lambda)$. It follows that $T_{4(p)}$ is the best estimator among the estimators $T_{im}{'}s, i = 1, 2, 3, 4$.

Tables 2-3 depicts the ranges of λ in which the suggested shrinkage estimators $T_{i(p)}{'}s, i = 1, 2, 3, 4$ are better than the corresponding usual unbiased estimators $t_i{'}s, i = 1, 2, 3, 4$ and the corresponding MMSE estimators $T_{im}{'}s, i = 1, 2, 3, 4$.

Tables 2-3 show that the proposed shrinkage estimators $T_{i(p)}{'}s, i = 1, 2, 3, 4$ are better than the corresponding usual unbiased estimators $t_i{'}s, i = 1, 2, 3, 4$ and the corresponding MMSE estimators $T_{im}{'}s, i = 1, 2, 3, 4$ for considerable ranges of λ .

It is further observed from Tables 2-3 that, although the class of estimators $T_{i(-1)}{'}s, i = 1, 2, 3, 4$; has the largest range of dominance, it offers smallest improvement compared with other competitors. The estimator $T_{i(2)}{'}s, i = 1, 2, 3, 4$ and $T_{i(-2)}{'}s, i = 1, 2, 3, 4$; offer large saving in MSE over , MMSE estimator T_{4m} but in a rather small range of λ . Thus it is interesting to mention that there is enough scope of selecting the suggested value θ_{20} of θ_2 to obtain better estimators which are useful in practice.

6. Conclusion

In this paper we have suggested some MMSE estimators and improved shrinkage estimators based on Chacko and Thomas (2008) estimators of the scale parameter θ_2 involved in (1.1) using ranked set sampling. We have obtained the expressions for biases and mean squared errors of the proposed estimators. It has been shown that the suggested estimators based on prior or guessed value θ_{20} are more efficient than those estimators including Chacko and Thomas (2008) estimators which do not utilize the guessed value θ_{20} , for a considerable range of the scale parameter θ_2 . Thus our recommendation is to use the suggested estimators in practice.

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Table 1 : The values of $e_i^{s\prime}, i = 1, 2, 3, 4$ for $p = -2$:

n	α	$\lambda = 1.00$				$\lambda = 1.20$ and $\lambda = 0.80$				$\lambda = 1.40$ and $\lambda = 0.60$			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
5	0.25	3.7552	3.7553	4.0281	4.2692	2.8696	2.8697	3.0263	3.1604	1.6806	1.6807	1.7332	1.7764
	0.50	3.3568	3.3572	3.7721	4.3556	2.5589	2.5591	2.7933	3.1009	1.4937	1.4937	1.5706	1.6634
	0.75	2.9911	2.9934	3.4495	4.4381	2.2704	2.2717	2.5251	3.0170	1.3178	1.3182	1.3997	1.5388
	1.00	2.6564	2.6647	3.0878	4.5158	2.0039	2.0086	2.2400	2.9068	1.1537	1.1553	1.2283	1.4050
10	0.25	1.9696	1.9691	2.1415	2.2375	1.7493	1.7489	1.8835	1.9574	1.3097	1.3095	1.3835	1.4229
	0.50	1.7487	1.7486	2.0010	2.2617	1.5508	1.5507	1.7460	1.9413	1.1577	1.1577	1.2632	1.3623
	0.75	1.5510	1.5536	1.8181	2.2844	1.3720	1.3741	1.5770	1.9163	1.0192	1.0203	1.1281	1.2917
	1.00	1.3746	1.3832	1.6132	2.3055	1.2115	1.2182	1.3931	1.8808	0.8935	0.8971	0.9885	1.2114
15	0.25	1.5388	1.5379	1.6812	1.7475	1.4273	1.4265	1.5490	1.6051	1.1724	1.1719	1.2533	1.2898
	0.50	1.3628	1.3638	1.5690	1.7604	1.2627	1.2637	1.4379	1.5970	1.0349	1.0355	1.1496	1.2492
	0.75	1.2067	1.2092	1.4217	1.7725	1.1162	1.1183	1.2977	1.5838	0.9112	0.9126	1.0286	1.2005
	1.00	1.0688	1.0776	1.2570	1.7835	0.9861	0.9936	1.1441	1.5646	0.8003	0.8052	0.9013	1.1434
20	0.25	1.3501	1.3492	1.4788	1.5330	1.2780	1.2771	1.3927	1.4407	1.1014	1.1008	1.1856	1.2202
	0.50	1.1942	1.1956	1.3791	1.5416	1.1295	1.1307	1.2936	1.4355	0.9716	0.9725	1.0906	1.1898
	0.75	1.0566	1.0583	1.2478	1.5496	0.9980	0.9996	1.1669	1.4269	0.8557	0.8569	0.9770	1.1528
	1.00	0.9355	0.9443	1.1011	1.5570	0.8819	0.8897	1.0276	1.4140	0.7526	0.7583	0.8562	1.1086

Table 1 : Continued...
for $p = -1$

n	α	$\lambda = 1.00$				$\lambda = 1.20$ and $\lambda = 0.80$				$\lambda = 1.40$ and $\lambda = 0.60$			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
5	0.25	1.0777	1.0777	1.1560	1.2252	1.0691	1.0691	1.1462	1.2141	1.0442	1.0442	1.1175	1.1821
	0.50	0.9634	0.9635	1.0826	1.2500	0.9556	0.9557	1.0728	1.2370	0.9331	0.9332	1.0445	1.1995
	0.75	0.8584	0.8591	0.9900	1.2737	0.8514	0.8520	0.9806	1.2583	0.8310	0.8316	0.9536	1.2142
	1.00	0.7624	0.7647	0.8862	1.2960	0.7559	0.7583	0.8775	1.2776	0.7374	0.7396	0.8526	1.2254
10	0.25	0.9793	0.9791	1.0648	1.1125	0.9754	0.9751	1.0601	1.1074	0.9638	0.9636	1.0465	1.0925
	0.50	0.8695	0.8694	0.9949	1.1245	0.8659	0.8659	0.9903	1.1186	0.8555	0.8555	0.9767	1.1013
	0.75	0.7712	0.7725	0.9040	1.1358	0.7680	0.7693	0.8996	1.1289	0.7585	0.7598	0.8867	1.1087
	1.00	0.6834	0.6877	0.8021	1.1463	0.6805	0.6848	0.7981	1.1381	0.6719	0.6761	0.7863	1.1142
15	0.25	0.9466	0.9460	1.0342	1.0750	0.9440	0.9435	1.0311	1.0717	0.9365	0.9360	1.0222	1.0620
	0.50	0.8383	0.8390	0.9652	1.0829	0.8360	0.8367	0.9621	1.0791	0.8293	0.8299	0.9532	1.0679
	0.75	0.7423	0.7438	0.8745	1.0903	0.7402	0.7418	0.8717	1.0859	0.7341	0.7356	0.8632	1.0728
	1.00	0.6575	0.6629	0.7732	1.0971	0.6556	0.6610	0.7706	1.0919	0.6500	0.6553	0.7629	1.0765
20	0.25	0.9302	0.9295	1.0188	1.0562	0.9283	0.9277	1.0166	1.0538	0.9228	0.9221	1.0099	1.0467
	0.50	0.8227	0.8237	0.9502	1.0621	0.8211	0.8220	0.9479	1.0594	0.8161	0.8170	0.9413	1.0511
	0.75	0.7279	0.7291	0.8597	1.0676	0.7264	0.7276	0.8576	1.0644	0.7219	0.7231	0.8513	1.0547
	1.00	0.6445	0.6506	0.7586	1.0727	0.6432	0.6492	0.7567	1.0689	0.6390	0.6450	0.7510	1.0575

Table 1 : Continued...
for $p = 1$

n	α	$\lambda = 1.00$				$\lambda = 1.20$ and $\lambda = 0.80$				$\lambda = 1.40$ and $\lambda = 0.60$			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
5	0.25	2.0789	2.0789	2.2300	2.3634	1.9086	1.9087	2.0353	2.1458	1.5322	1.5322	1.6128	1.6814
	0.50	1.8584	1.8586	2.0883	2.4113	1.7047	1.7049	1.8962	2.1588	1.3659	1.3660	1.4861	1.6427
	0.75	1.6559	1.6572	1.9097	2.4570	1.5167	1.5178	1.7269	2.1625	1.2113	1.2120	1.3417	1.5907
	1.00	1.4706	1.4752	1.7094	2.5000	1.3441	1.3479	1.5408	2.1552	1.0684	1.0708	1.1891	1.5244
10	0.25	1.2646	1.2643	1.3750	1.4366	1.2336	1.2333	1.3384	1.3967	1.1492	1.1489	1.2396	1.2895
	0.50	1.1228	1.1227	1.2847	1.4521	1.0949	1.0949	1.2484	1.4059	1.0191	1.0190	1.1508	1.2833
	0.75	0.9958	0.9975	1.1673	1.4667	0.9706	0.9722	1.1328	1.4126	0.9020	0.9034	1.0405	1.2719
	1.00	0.8825	0.8881	1.0358	1.4803	0.8595	0.8647	1.0042	1.4165	0.7970	0.8015	0.9199	1.2545
15	0.25	1.1076	1.1070	1.2101	1.2579	1.0920	1.0914	1.1915	1.2378	1.0478	1.0473	1.1391	1.1813
	0.50	0.9809	0.9817	1.1294	1.2672	0.9669	0.9677	1.1109	1.2439	0.9273	0.9280	1.0588	1.1790
	0.75	0.8686	0.8704	1.0233	1.2758	0.8559	0.8577	1.0058	1.2487	0.8200	0.8216	0.9566	1.1737
	1.00	0.7693	0.7757	0.9048	1.2838	0.7577	0.7639	0.8888	1.2518	0.7249	0.7305	0.8440	1.1647
20	0.25	1.0416	1.0409	1.1409	1.1827	1.0314	1.0307	1.1286	1.1696	1.0019	1.0012	1.0933	1.1317
	0.50	0.9213	0.9224	1.0640	1.1894	0.9121	0.9132	1.0518	1.1741	0.8856	0.8866	1.0167	1.1306
	0.75	0.8151	0.8165	0.9626	1.1955	0.8068	0.8081	0.9511	1.1777	0.7828	0.7841	0.9179	1.1273
	1.00	0.7218	0.7285	0.8495	1.2012	0.7141	0.7208	0.8390	1.1802	0.6922	0.6984	0.8088	1.1215

small **Table 1** : Continued...
for $p = 2$

n	α	$\lambda = 1.00$				$\lambda = 1.20$ and $\lambda = 0.80$				$\lambda = 1.40$ and $\lambda = 0.60$			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
5	0.25	116.9370	116.9403	125.4378	132.9438	4.2457	4.2458	4.2562	4.2644	1.0911	1.0911	1.0918	1.0924
	0.50	104.5335	104.5452	117.4651	135.6337	3.7572	3.7572	3.7721	3.7884	0.9653	0.9653	0.9663	0.9674
	0.75	93.1428	93.2150	107.4192	138.2036	3.2909	3.2910	3.3064	3.3292	0.8451	0.8451	0.8461	0.8476
	1.00	82.7206	82.9793	96.1538	140.6250	2.8519	2.8522	2.8657	2.8929	0.7319	0.7319	0.7328	0.7346
10	0.25	4.5882	4.5870	4.9885	5.2121	2.5979	2.5976	2.7216	2.7868	1.1289	1.1288	1.1516	1.1631
	0.50	4.0735	4.0733	4.6612	5.2685	2.2936	2.2935	2.4689	2.6294	0.9925	0.9925	1.0240	1.0506
	0.75	3.6129	3.6191	4.2353	5.3215	2.0148	2.0167	2.1946	2.4542	0.8658	0.8662	0.8974	0.9380
	1.00	3.2020	3.2221	3.7579	5.3706	1.7606	1.7667	1.9166	2.2631	0.7491	0.7502	0.7759	0.8272
15	0.25	2.4172	2.4158	2.6409	2.7451	1.8605	1.8597	1.9903	2.0489	1.1003	1.1001	1.1445	1.1636
	0.50	2.1407	2.1424	2.4646	2.7654	1.6424	1.6434	1.8267	1.9868	0.9671	0.9675	1.0282	1.0771
	0.75	1.8956	1.8995	2.2333	2.7843	1.4464	1.4487	1.6350	1.9121	0.8454	0.8462	0.9065	0.9857
	1.00	1.6789	1.6928	1.9746	2.8017	1.2707	1.2786	1.4331	1.8239	0.7348	0.7374	0.7863	0.8910
20	0.25	1.8246	1.8233	1.9985	2.0718	1.5489	1.5480	1.6724	1.7235	1.0658	1.0653	1.1228	1.1456
	0.50	1.6138	1.6157	1.8638	2.0835	1.3670	1.3684	1.5422	1.6896	0.9371	0.9377	1.0162	1.0782
	0.75	1.4279	1.4303	1.6863	2.0943	1.2050	1.2066	1.3839	1.6473	0.8206	0.8214	0.8999	1.0042
	1.00	1.2643	1.2762	1.4881	2.1042	1.0610	1.0694	1.2143	1.5954	0.7157	0.7195	0.7824	0.9247

Table 2 :The Ranges of λ in which the suggested shrinkage estimators $T'_{i(p)}s, i = 1, 2, 3, 4$ are better than the corresponding usual unbiased estimators $t'_i s, i = 1, 2, 3, 4$ given by Chacko and Thomas (2008):

n	α	$p = -2$				$p = -1$			
		t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
5	0.25	(0.33,1.67)	(0.33,1.67)	(0.36,1.64)	(0.38,1.62)	(0,2.48)	(0,2.48)	(0,2.43)	(0,2.39)
	0.50	(0.34,1.66)	(0.34,1.66)	(0.38,1.62)	(0.42,1.58)	(0,2.47)	(0,2.47)	(0,2.39)	(0,2.29)
	0.75	(0.34,1.66)	(0.34,1.66)	(0.39,1.61)	(0.46,1.54)	(0,2.46)	(0,2.46)	(0,2.36)	(0,2.20)
	1.00	(0.35,1.65)	(0.35,1.65)	(0.40,1.60)	(0.50,1.50)	(0,2.44)	(0,2.44)	(0,2.34)	(0,2.11)
10	0.25	(0.40,1.60)	(0.40,1.60)	(0.42,1.58)	(0.44,1.56)	(0,2.45)	(0,2.45)	(0,2.39)	(0,2.36)
	0.50	(0.40,1.60)	(0.40,1.60)	(0.44,1.56)	(0.47,1.53)	(0,2.44)	(0,2.44)	(0,2.34)	(0,2.26)
	0.75	(0.41,1.59)	(0.41,1.59)	(0.45,1.55)	(0.51,1.49)	(0,2.42)	(0,2.42)	(0,2.31)	(0,2.17)
	1.00	(0.42,1.58)	(0.42,1.58)	(0.46,1.54)	(0.55,1.45)	(0,2.40)	(0,2.39)	(0,2.29)	(0,2.08)
15	0.25	(0.42,1.58)	(0.42,1.58)	(0.45,1.55)	(0.46,1.54)	(0,2.43)	(0,2.43)	(0,2.37)	(0,2.35)
	0.50	(0.42,1.58)	(0.43,1.57)	(0.46,1.54)	(0.49,1.51)	(0,2.42)	(0,2.42)	(0,2.33)	(0,2.25)
	0.75	(0.43,1.57)	(0.43,1.57)	(0.48,1.52)	(0.53,1.47)	(0,2.41)	(0,2.41)	(0,2.30)	(0,2.16)
	1.00	(0.44,1.56)	(0.44,1.56)	(0.48,1.52)	(0.57,1.43)	(0,2.38)	(0,2.38)	(0,2.28)	(0,2.07)
20	0.25	(0.43,1.57)	(0.43,1.57)	(0.46,1.54)	(0.47,1.53)	(0,2.43)	(0,2.43)	(0,2.36)	(0,2.34)
	0.50	(0.44,1.56)	(0.44,1.56)	(0.48,1.52)	(0.50,1.50)	(0,2.42)	(0,2.42)	(0,2.32)	(0,2.25)
	0.75	(0.44,1.56)	(0.44,1.56)	(0.49,1.51)	(0.54,1.46)	(0,2.40)	(0,2.40)	(0,2.29)	(0,2.16)
	1.00	(0.45,1.55)	(0.46,1.54)	(0.50,1.50)	(0.58,1.42)	(0,2.38)	(0,2.37)	(0,2.27)	(0,2.07)

Table 2 : Continued... :

n	α	$p=1$				$p=2$			
		t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
5	0.25	(0.11,1.89)	(0.11,1.89)	(0.14,1.86)	(0.16,1.84)	(0.52,1.48)	(0.52,1.48)	(0.53,1.47)	(0.55,1.45)
	0.50	(0.11,1.89)	(0.11,1.89)	(0.16,1.84)	(0.22,1.78)	(0.52,1.48)	(0.52,1.48)	(0.55,1.45)	(0.58,1.42)
	0.75	(0.12,1.88)	(0.12,1.88)	(0.18,1.82)	(0.28,1.72)	(0.52,1.48)	(0.52,1.48)	(0.56,1.44)	(0.61,1.39)
	1.00	(0.13,1.87)	(0.13,1.87)	(0.19,1.81)	(0.33,1.67)	(0.53,1.47)	(0.53,1.47)	(0.56,1.44)	(0.64,1.36)
10	0.25	(0.05,1.95)	(0.05,1.95)	(0.09,1.91)	(0.11,1.89)	(0.51,1.49)	(0.51,1.49)	(0.53,1.47)	(0.54,1.46)
	0.50	(0.06,1.94)	(0.06,1.94)	(0.12,1.88)	(0.17,1.83)	(0.51,1.49)	(0.51,1.49)	(0.54,1.46)	(0.57,1.43)
	0.75	(0.07,1.93)	(0.07,1.93)	(0.14,1.86)	(0.23,1.77)	(0.52,1.49)	(0.52,1.48)	(0.55,1.45)	(0.60,1.40)
	1.00	(0.08,1.92)	(0.09,1.91)	(0.15,1.85)	(0.29,1.71)	(0.52,1.48)	(0.52,1.48)	(0.56,1.44)	(0.63,1.37)
15	0.25	(0.04,1.96)	(0.04,1.96)	(0.08,1.92)	(0.10,1.90)	(0.50,1.50)	(0.50,1.50)	(0.52,1.48)	(0.53,1.47)
	0.50	(0.04,1.96)	(0.04,1.96)	(0.11,1.89)	(0.16,1.84)	(0.50,1.50)	(0.50,1.50)	(0.53,1.47)	(0.56,1.44)
	0.75	(0.05,1.95)	(0.05,1.95)	(0.13,1.87)	(0.22,1.78)	(0.51,1.50)	(0.51,1.49)	(0.54,1.46)	(0.59,1.41)
	1.00	(0.07,1.93)	(0.07,1.93)	(0.14,1.86)	(0.28,1.72)	(0.51,1.49)	(0.51,1.49)	(0.55,1.45)	(0.62,1.38)
20	0.25	(0.03,1.97)	(0.03,1.97)	(0.07,1.93)	(0.09,1.91)	(0.49,1.51)	(0.49,1.51)	(0.51,1.49)	(0.52,1.48)
	0.50	(0.03,1.97)	(0.04,1.96)	(0.10,1.90)	(0.15,1.85)	(0.49,1.51)	(0.49,1.51)	(0.53,1.47)	(0.55,1.45)
	0.75	(0.05,1.95)	(0.05,1.95)	(0.12,1.88)	(0.21,1.79)	(0.50,1.50)	(0.50,1.50)	(0.54,1.46)	(0.59,1.41)
	1.00	(0.06,1.94)	(0.07,1.93)	(0.14,1.86)	(0.27,1.73)	(0.51,1.49)	(0.51,1.49)	(0.55,1.45)	(0.62,1.38)

Table 3 :The Ranges of λ in which the suggested shrinkage estimators $T'_{i(p)}s, i = 1, 2, 3, 4$ are better than the corresponding MMSE estimators $T'_{im}s, i = 1, 2, 3, 4$:

n	α	$p = -2$				$p = -1$			
		T_{1m}	T_{2m}	T_{3m}	T_{4m}	T_{1m}	T_{2m}	T_{3m}	T_{4m}
5	0.25	(0.36,1.64)	(0.36,1.64)	(0.35,1.65)	(0.35,1.65)	(0,2)	(0,2)	(0,2.03)	(0,2.06)
	0.50	(0.36,1.64)	(0.36,1.64)	(0.35,1.65)	(0.34,1.66)	(0,2)	(0,2)	(0,2.05)	(0,2.11)
	0.75	(0.36,1.64)	(0.36,1.64)	(0.36,1.64)	(0.34,1.66)	(0,2)	(0,2)	(0,2.06)	(0,2.15)
	1.00	(0.37,1.63)	(0.37,1.63)	(0.36,1.64)	(0.34,1.66)	(0,2)	(0,2)	(0,2.06)	(0,2.18)
10	0.25	(0.38,1.62)	(0.38,1.62)	(0.38,1.62)	(0.37,1.63)	(0,2)	(0,2)	(0,2.04)	(0,2.06)
	0.50	(0.38,1.62)	(0.38,1.62)	(0.38,1.62)	(0.37,1.63)	(0,2)	(0,2)	(0,2.06)	(0,2.11)
	0.75	(0.39,1.61)	(0.39,1.61)	(0.38,1.62)	(0.37,1.63)	(0,2)	(0,2)	(0,2.07)	(0,2.14)
	1.00	(0.40,1.60)	(0.40,1.60)	(0.39,1.61)	(0.38,1.62)	(0,2)	(0,2)	(0,2.06)	(0,2.17)
15	0.25	(0.39,1.61)	(0.39,1.61)	(0.38,1.62)	(0.38,1.62)	(0,2)	(0,2)	(0,2.04)	(0,2.06)
	0.50	(0.39,1.61)	(0.39,1.61)	(0.38,1.62)	(0.38,1.62)	(0,2)	(0,2)	(0,2.06)	(0,2.10)
	0.75	(0.40,1.60)	(0.40,1.60)	(0.39,1.61)	(0.38,1.62)	(0,2)	(0,2)	(0,2.07)	(0,2.14)
	1.00	(0.41,1.59)	(0.40,1.60)	(0.40,1.60)	(0.39,1.61)	(0,2)	(0,2)	(0,2.06)	(0,2.16)
20	0.25	(0.39,1.61)	(0.39,1.61)	(0.39,1.61)	(0.39,1.61)	(0,2)	(0,2)	(0,2.04)	(0,2.06)
	0.50	(0.39,1.61)	(0.39,1.61)	(0.39,1.61)	(0.38,1.62)	(0,2)	(0,2)	(0,2.06)	(0,2.10)
	0.75	(0.40,1.60)	(0.40,1.60)	(0.39,1.61)	(0.39,1.61)	(0,2)	(0,2)	(0,2.07)	(0,2.14)
	1.00	(0.41,1.59)	(0.41,1.59)	(0.40,1.60)	(0.39,1.61)	(0,2)	(0,2)	(0,2.06)	(0,2.16)

Table 3 : Continued... :

n	α	$p=1$				$p=2$			
		T_{1m}	T_{2m}	T_{3m}	T_{4m}	T_{1m}	T_{2m}	T_{3m}	T_{4m}
5	0.25	(0.23,1.77)	(0.23,1.77)	(0.22,1.78)	(0.22,1.78)	(0.56,1.44)	(0.56,1.44)	(0.56,1.44)	(0.55,1.45)
	0.50	(0.23,1.77)	(0.23,1.77)	(0.22,1.78)	(0.21,1.79)	(0.56,1.44)	(0.56,1.44)	(0.56,1.44)	(0.55,1.45)
	0.75	(0.24,1.76)	(0.24,1.76)	(0.22,1.78)	(0.20,1.80)	(0.56,1.44)	(0.56,1.44)	(0.56,1.44)	(0.55,1.45)
	1.00	(0.25,1.75)	(0.25,1.75)	(0.23,1.77)	(0.20,1.80)	(0.57,1.43)	(0.57,1.43)	(0.56,1.44)	(0.55,1.45)
10	0.25	(0.18,1.82)	(0.18,1.82)	(0.17,1.83)	(0.17,1.83)	(0.53,1.47)	(0.53,1.47)	(0.53,1.47)	(0.53,1.47)
	0.50	(0.18,1.82)	(0.18,1.82)	(0.17,1.83)	(0.16,1.84)	(0.54,1.46)	(0.54,1.46)	(0.53,1.47)	(0.53,1.47)
	0.75	(0.19,1.81)	(0.19,1.81)	(0.17,1.83)	(0.15,1.85)	(0.54,1.46)	(0.54,1.46)	(0.54,1.46)	(0.53,1.47)
	1.00	(0.20,1.80)	(0.20,1.80)	(0.18,1.82)	(0.15,1.85)	(0.55,1.45)	(0.55,1.45)	(0.54,1.46)	(0.54,1.46)
15	0.25	(0.17,1.83)	(0.17,1.83)	(0.15,1.85)	(0.15,1.85)	(0.52,1.48)	(0.52,1.48)	(0.52,1.48)	(0.52,1.48)
	0.50	(0.17,1.83)	(0.17,1.83)	(0.15,1.85)	(0.14,1.86)	(0.52,1.48)	(0.52,1.48)	(0.52,1.48)	(0.52,1.48)
	0.75	(0.18,1.82)	(0.18,1.82)	(0.16,1.84)	(0.14,1.86)	(0.53,1.47)	(0.53,1.47)	(0.52,1.48)	(0.52,1.48)
	1.00	(0.19,1.81)	(0.18,1.82)	(0.17,1.83)	(0.14,1.86)	(0.54,1.46)	(0.53,1.47)	(0.53,1.47)	(0.53,1.47)
20	0.25	(0.16,1.84)	(0.16,1.84)	(0.15,1.85)	(0.14,1.86)	(0.51,1.49)	(0.51,1.49)	(0.51,1.49)	(0.51,1.49)
	0.50	(0.16,1.84)	(0.16,1.84)	(0.14,1.86)	(0.13,1.87)	(0.52,1.48)	(0.52,1.48)	(0.51,1.49)	(0.51,1.49)
	0.75	(0.17,1.83)	(0.17,1.83)	(0.15,1.85)	(0.13,1.87)	(0.52,1.48)	(0.52,1.48)	(0.52,1.48)	(0.51,1.49)
	1.00	(0.18,1.82)	(0.18,1.82)	(0.16,1.84)	(0.13,1.87)	(0.53,1.47)	(0.53,1.47)	(0.52,1.48)	(0.52,1.48)