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The product of generalized superderivations on a prime superalgebra

He Yuan* and Yu Wang†

Abstract

In the paper, we extend the definition of generalized derivations to superalgebras and prove that a generalized superderivation g on a prime superalgebra A is represented as $g(x) = ax + d(x)$ for all $x \in A$, where a is an element of Q_{mr} (the maximal right ring of quotients of A) and d is a superderivation on A. Using the result we study two generalized superderivations when their product is also a generalized superderivation on a prime superalgebra A.

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1. Introduction

Let R be a prime ring. According to Hvala [9] an additive mapping $g : R \to R$ is said to be a generalized derivation of R if there exists a derivation δ of R such that $g(xy) = g(x)y + x\delta(y)$ for all $x, y \in R$. In [14] Lee proved that every generalized derivation of A can be uniquely extended to Q_{mr} and there exists an element $a \in Q_{mr}$ such that $g(x) = ax + \delta(x)$ for all $x \in R$.

The study of the product of derivations in prime rings was initiated by Posner [18]. He proved that the product of two nonzero derivations can not be a derivation on a prime ring of characteristic not 2. Later a number of authors studied the problem in several ways (see [2], [4], [5], [9], [10], [12], [13], and [15]). Hvala [9] studied two generalized derivations f_1, f_2 when the product is also a generalized derivation on a prime ring R of characteristic not 2 in 1998. In 2001 Lee [13] gave a description of Hvala's Theorem without the assumption of char$R \neq 2$. In 2004 Fošner [5] extended Posner's Theorem to prime superalgebras.

*Department of Mathematics Jilin University Changchun 130012, P. R. of China and Department of Mathematics Jilin Normal University Siping 136000, P. R. of China
Email: yuanhe1983@126.com

†Department of Mathematics Shanghai Normal University Shanghai 200234, P. R. of China
Email:ywang2004@126.com

Superalgebras first appeared in physics, in the Theory of Supersymmetry, to create an algebraic structure representing the behavior of the subatomic particles known as bosons and fermions ([11]). Recently there has been a considerable authors who are interested in superalgebras. They extended many results of rings to superalgebras (see [3], [5], [6], [7], [8], [11], [16], [17] and [19]).

In Section 3, we will extend the definition of generalized derivations to superalgebras and prove that every generalized superderivation of a prime superalgebra A can be extended to Q_{mr} (the maximal right ring of quotients of A). Further, we will prove that a generalized superderivation of a prime superalgebra is a sum of a left multiplication mapping and a superderivation. Using the result we will study two generalized superderivations when their product is also a generalized superderivation on a prime superalgebra. As a result, Fošner's theorem [5, Theorem 4.1] is the special case of the main theorem of the paper.

2. preliminaries

Let Φ be a commutative ring with $\frac{1}{2} \in \Phi$. An associative algebra A over Φ is said to be an associative superalgebra if there exist two Φ-submodules A_0 and A_1 of A such that $A = A_0 \bigoplus A_1$ and $A_iA_j \subseteq A_{i+j}$, $i, j \in Z_2$. A superalgebra is called trivial if $A_1 = 0$. The elements of A_i are homogeneous of degree i and we write $|a_i| = i$ for all $a_i \in A_i$. We define $[a, b]_s = ab - (-1)^{|a||b|}ba$ for all $a, b \in A_0 \cup A_1$. Thus, $[a, b]_s = [a_0, b_0]_s + [a_1, b_0]_s + [a_0, b_1]_s + [a_1, b_1]_s$, where $a = a_0 + a_1$, $b = b_0 + b_1$ and $a_i, b_i \in A_i$ for $i = 0, 1$. It follows that $[a, b]_s = [a, b]$ if one of the elements a and b is homogeneous of degree 0. Let $k \in \{0, 1\}$. A superderivation of degree k is actually a Φ-linear mapping $d_k : A \to A$ which satisfies $d_k(A_i) \subseteq A_{k+i}$ for $i \in Z_2$ and $d_k(ab) = d_k(a)b+(-1)^{k|a|}ad_k(b)$ for all $a, b \in A_0 \cup A_1$. If $d = d_0 + d_1$, then d is a superderivation on A. For example, for $a = a_0 + a_1 \in A$ the mapping $\mathrm{ad}_s(a)(x) = [a, x]_s = [a_0, x]_s + [a_1, x]_s$ is a superderivation, which is called the inner superderivation induced by a. For a superalgebra A, we define $\sigma : A \to A$ by $(a_0 + a_1)^\sigma = a_0 - a_1$, then σ is an automorphism of A such that $\sigma^2 = 1$. On the other hand, for an algebra A, if there exists an automorphism σ of A such that $\sigma^2 = 1$, then A becomes a superalgebra $A = A_0 \bigoplus A_1$, where $A_i = \{x \in A | x^\sigma = (-1)^i x\}$, $i = 0, 1$. Clearly a superderivation d of degree 1 is a σ-derivation, i.e., it satisfies $d(ab) = d(a)b + a^\sigma d(b)$ for all $a, b \in A$. A superalgebra A is called a prime superalgebra if and only if $aAb = 0$ implies $a = 0$ or $b = 0$, where at least one of the elements a and b is homogeneous. The knowledge of superalgebras refers to [3], [5], [6], [7], [8], [16], [17] and [19].

In [17] Montaner obtained that a prime superalgebra A is not necessarily a prime algebra but a semiprime algebra. Hence one can define the maximal right ring of quotients Q_{mr} of A, and the useful properties of Q_{mr} can be found in [1]. By [1, proposition 2.5.3] σ can be uniquely extended to Q_{mr} such that $\sigma^2 = 1$. Therefore Q_{mr} is also a superalgebra. Further, we can get that Q_{mr} is a prime superalgebra.

3. the product of generalized superderivations

Firstly, we extend the definition of generalized derivations to superalgebras.

3.1. Definition. Let A be a superalgebra. For $i \in \{0, 1\}$, a Φ-linear mapping $g_i : A \to A$ is called a generalized superderivation of degree i if $g_i(A_j) \subseteq A_{i+j}$, $j \in Z_2$, and $g_i(xy) = g_i(x)y + (-1)^{i|x|}xd_i(y)$ for all $x, y \in A_0 \cup A_1$, where d_i is a superderivation of degree i on A. If $g = g_0 + g_1$, then g is called a generalized superderivation on A.

Let A be a prime superalgebra and $Q = Q_{mr}$ be the maximal right ring of quotients of A. Next, we prove that a generalized superderivation of a prime superalgebra is a sum of a left multiplication mapping and a superderivation. By [20, proposition 2] we have every σ-derivation d of a semiprime ring A can be uniquely extended to a σ-derivation of Q.

3.2. Theorem. *Let A be a prime superalgebra and $g : A \to A$ be a generalized superderivation. Then g can be extended to Q and there exist an element $a \in Q$ and a superderivation d of A such that $g(x) = ax + d(x)$ for all $x \in A$, where both a and d are determined by g uniquely.*

Proof. To prove that the generalized superderivation g on a prime superalgebra A can be extended to Q, it suffices to prove that g_0 and g_1 can be extended to Q, respectively. The generalized superderivation of degree 1 g_1 is represented as $g_1(xy) = g_1(x)y + x^\sigma d_1(y)$ for all $x, y \in A$, where d_1 is a superderivation of degree 1 on A. Note that $d_1(xy) = d_1(x)y + x^\sigma d_1(y)$. So combining the two equations we have $(g_1 - d_1)(xy) = (g_1 - d_1)(x)y$. Let $g_1 - d_1 = f$. Clearly f is a right A-module mapping. Then there exists $a_1 \in Q$ such that $f(x) = a_1 x$. So $g_1(x) = a_1 x + d_1(x)$ for all $x \in A$. Since d_1 can be extended to Q, then it follows that g_1 can be extended to Q. It is easy to prove that $g_0(x) = a_0 x + d_0(x)$ and g_0 can be extended to Q similarly, where a_0 is an element of Q and d_0 is a superderivation of degree 0 on A. So g can be extended to Q. Clearly $a_i \in Q_i$, $i \in \{0, 1\}$. Let $a = a_0 + a_1$ and $d = d_0 + d_1$. Then $g(x) = g_0(x) + g_1(x) = a_0 x + d_0(x) + a_1 x + d_1(x) = ax + d(x)$ for all $x \in A$, where a is an element of Q and d is a superderivation of A.

Now we claim both a and d are determined by g uniquely. It suffices to prove that $a = 0$ and $d = 0$ when $g = 0$. Since $g = 0$, we have $g_0 = g_1 = 0$. By $g_1 = 0$, we obtain $0 = g_1(yr) = a_1 yr + d_1(yr) = a_1 yr + d_1(y)r + y^\sigma d_1(r) = g_1(y)r + y^\sigma d_1(r) = y^\sigma d_1(r)$ for all $y, r \in A$. Then $A^\sigma d_1(A) = 0$. So $Ad_1(A) = 0$. Clearly $d_1(A) = 0$. Since $g_1(A) = 0$, it follows that $a_1 A = 0$. Hence $a_1 = 0$. Similarly we can prove the case when $g_0 = 0$. So $a = 0$ and $d = 0$. □

Next, we give two results which are used in the proof of the main result.

3.3. Lemma. *Let A be a prime superalgebra. If A satisfies*

$$([a_0, x] + d_0(x))yk_0(z) + ([b_0, x] + k_0(x))yd_0(z) = 0 \qquad \textit{for all } x, y, z \in A, \tag{3.1}$$

where a_0, $b_0 \in Q_0$ and both d_0 and k_0 are superderivations of degree 0 on A. Then one of the following cases is true:

(i) There exists $0 \neq \mu \in C_0$ such that $\mu k_0(x) + d_0(x) = 0$;

(ii) $[a_0, x] + d_0(x) = 0$;

(iii) $[b_0, x] + k_0(x) = 0$

for all $x \in A$.

Proof. Let $d_0 = k_0 = 0$. Clearly there exists $0 \neq \mu \in C_0$ such that $\mu k_0(x) + d_0(x) = 0$. Hence (i) is true.

Next we assume either $d_0 \neq 0$ or $k_0 \neq 0$. By [5, Theorem 3.3] there exist λ_1 and λ_2 not all zero such that $\lambda_1([a_0, x] + d_0(x)) + \lambda_2([b_0, x] + k_0(x)) = 0$. Let $\lambda_1 = \lambda_{10} + \lambda_{11}$ and $\lambda_2 = \lambda_{20} + \lambda_{21}$. Then $\lambda_{10}([a_0, x] + d_0(x)) + \lambda_{11}([a_0, x] + d_0(x)) + \lambda_{20}([b_0, x] + k_0(x)) + \lambda_{21}([b_0, x] + k_0(x)) = 0$ for all $x \in A$, where $\lambda_{10}, \lambda_{20} \in C_0$, $\lambda_{11}, \lambda_{21} \in C_1$. By $A_0 \cap A_1 = 0$, we have

$$\lambda_{11}([a_0, x_0] + d_0(x_0)) + \lambda_{21}([b_0, x_0] + k_0(x_0)) = 0 \qquad \text{for all } x_0 \in A_0, \tag{3.2}$$

$$\lambda_{11}([a_0, x_1] + d_0(x_1)) + \lambda_{21}([b_0, x_1] + k_0(x_1)) = 0 \qquad \text{for all } x_1 \in A_1. \tag{3.3}$$

Using (3.2) and (3.3) we obtain

$$\lambda_{11}([a_0, x] + d_0(x)) + \lambda_{21}([b_0, x] + k_0(x)) = 0 \qquad \text{for all } x \in A. \tag{3.4}$$

We proceed by dividing three cases. Only one of λ_{11} and λ_{21} is nonzero. If $\lambda_{21} \neq 0$, then $[b_0, x] + k_0(x) = 0$. If $\lambda_{11} \neq 0$, then $[a_0, x] + d_0(x) = 0$. Hence either (ii) or (iii) is true.

Both $\lambda_{11} \neq 0$ and $\lambda_{21} \neq 0$. By (3.4) and [5, Lemma 3.1] we arrive at $[a_0, x] + d_0(x) = \lambda([b_0, x] + k_0(x))$, where $\lambda = -\lambda_{11}^{-1}\lambda_{21} \neq 0$. Using (3.1) we get $\lambda([b_0, x] + k_0(x))yk_0(z) + ([b_0, x] + k_0(x))yd_0(z) = 0$. That is, $([b_0, x] + k_0(x))y(\lambda k_0(z) + d_0(z)) = 0$. If there exists $z \in A$ such that $\lambda k_0(z) + d_0(z) \neq 0$, then $[b_0, x] + k_0(x) = 0$ for all $x \in A_0 \cup A_1$. It follows that $[b_0, x] + k_0(x) = 0$ for all $x \in A$. Hence either (i) or (iii) is true. Similarly, when $\rho([a_0, x] + d_0(x)) = [b_0, x] + k_0(x)$, where $\rho = -\lambda_{21}^{-1}\lambda_{11} \neq 0$, we have either (i) or (ii) is true by using (3.1) again.

When $\lambda_{11} = \lambda_{21} = 0$, i.e., $\lambda_1, \lambda_2 \in C_0$. If one of λ_1 and λ_2 is zero, then either (ii) or (iii) is true. If both λ_1 and λ_2 are nonzero, the proof is similar to the above paragraph. □

Similar to the proof of Lemma 3.3, we can get the following result.

3.4. Lemma. *Let A be a prime superalgebra. If A satisfies*

$$([a_1, x]_s + d_1(x))yk_1(z) - ([b_1, x]_s + k_1(x))yd_1(z) = 0 \qquad \text{for all } x, y, z \in A,$$

where a_1, $b_1 \in Q_1$ and both d_1 and k_1 are superderivations of degree 1 on A. Then one of the following cases is true:

(i) There exists $0 \neq \nu \in C_0$ such that $\nu k_1(x) + d_1(x) = 0$;

(ii) $[a_1, x]_s + d_1(x) = 0$;

(iii) $[b_1, x]_s + k_1(x) = 0$

for all $x \in A$.

Now, we are in a position to give the main result of this paper.

3.5. Theorem. *Let A be a prime superalgebra and let $f = a + d$ and $g = b + k$ be two nonzero generalized superderivations on A, where $a, b \in Q$ and both d and k are superderivations on A. If fg is also a generalized superderivation on A. Then one of the following cases is true:*

(i) There exists $0 \neq \omega \in C_0$ such that $\omega k_j(x) + d_j(x) = 0$;

(ii) $[a_i, x]_s + d_i(x) = 0$;

(iii) $[b_i, x]_s + k_i(x) = 0$

for all $x \in A$, where $i, j \in \{0, 1\}$, $a_i, b_i \in Q_i$ and both d_i and k_i are superderivations of degree i on A, as well as d_j and k_j.

Proof. According to Theorem 3.2 we assume $h(x) = fg(x) = cx + l(x)$ for all $x \in A$, where $c \in Q$ and l is a superderivation on A, then

$$\begin{aligned} fg(x) =& a(bx + k(x)) + d(bx + k(x)) \\ =& abx + ak(x) + d_0(b)x + bd_0(x) + d_1(b)x + b^\sigma d_1(x) + d_0k(x) + d_1k(x). \end{aligned}$$

Hence

$$\begin{aligned} c =& ab + d_0(b) + d_1(b) = ab + d(b), \\ l(x) =& ak(x) + bd_0(x) + b^\sigma d_1(x) + d_0k(x) + d_1k(x), \\ l_0(x) =& a_0k_0(x) + a_1k_1(x) + b_0d_0(x) - b_1d_1(x) + d_0k_0(x) + d_1k_1(x), \\ l_1(x) =& a_1k_0(x) + a_0k_1(x) + b_1d_0(x) + b_0d_1(x) + d_0k_1(x) + d_1k_0(x). \end{aligned}$$

On the one hand we get

$$\begin{aligned}l_0(xy) =&a_0k_0(xy) + a_1k_1(xy) + b_0d_0(xy) - b_1d_1(xy) + d_0k_0(xy) + d_1k_1(xy)\\ =&a_0k_0(x)y + a_0xk_0(y) + a_1k_1(x)y + a_1x^\sigma k_1(y)\\ &+ b_0d_0(x)y + b_0xd_0(y) - b_1d_1(x)y - b_1x^\sigma d_1(y)\\ &+ d_0k_0(x)y + k_0(x)d_0(y) + d_0(x)k_0(y) + xd_0k_0(y)\\ &+ d_1k_1(x)y + k_1(x)^\sigma d_1(y) + d_1(x^\sigma)k_1(y) + xd_1k_1(y)\end{aligned}$$

and on the other hand we get

$$\begin{aligned}l_0(xy) =&a_0k_0(x)y + a_1k_1(x)y + b_0d_0(x)y - b_1d_1(x)y + d_0k_0(x)y + d_1k_1(x)y\\ &+ x[a_0k_0(y) + a_1k_1(y) + b_0d_0(y) - b_1d_1(y) + d_0k_0(y) + d_1k_1(y)].\end{aligned}$$

Combining the two equations we have

$$\begin{aligned}0 =&[a_0, x]k_0(y) + a_1x^\sigma k_1(y) - xa_1k_1(y) + [b_0, x]d_0(y) - b_1x^\sigma d_1(y)\\ &+ xb_1d_1(y) + k_0(x)d_0(y) + d_0(x)k_0(y) + k_1(x)^\sigma d_1(y) - d_1(x)^\sigma k_1(y).\end{aligned} \tag{3.5}$$

In particular, replacing y by yz in (3.5) we get

$$\begin{aligned}0 =&[a_0, x]k_0(yz) + a_1x^\sigma k_1(yz) - xa_1k_1(yz) + [b_0, x]d_0(yz) - b_1x^\sigma d_1(yz)\\ &+ xb_1d_1(yz) + k_0(x)d_0(yz) + d_0(x)k_0(yz) + k_1(x)^\sigma d_1(yz) - d_1(x)^\sigma k_1(yz).\end{aligned}$$

Extending the identity above we arrive at

$$\begin{aligned}0 =&[a_0, x]k_0(y)z + [a_0, x]yk_0(z) + a_1x^\sigma k_1(y)z + a_1x^\sigma y^\sigma k_1(z)\\ &- xa_1k_1(y)z - xa_1y^\sigma k_1(z) + [b_0, x]d_0(y)z + [b_0, x]yd_0(z)\\ &- b_1x^\sigma d_1(y)z - b_1x^\sigma y^\sigma d_1(z) + xb_1d_1(y)z + xb_1y^\sigma d_1(z)\\ &+ k_0(x)d_0(y)z + k_0(x)yd_0(z) + d_0(x)k_0(y)z + d_0(x)yk_0(z)\\ &+ k_1(x)^\sigma d_1(y)z + k_1(x)^\sigma y^\sigma d_1(z) - d_1(x)^\sigma k_1(y)z - d_1(x)^\sigma y^\sigma k_1(z).\end{aligned}$$

Using (3.5) we have

$$\begin{aligned}0 =&[a_0, x]yk_0(z) + a_1x^\sigma y^\sigma k_1(z) - xa_1y^\sigma k_1(z) + [b_0, x]yd_0(z) - b_1x^\sigma y^\sigma d_1(z)\\ &+ xb_1y^\sigma d_1(z) + k_0(x)yd_0(z) + d_0(x)yk_0(z) + k_1(x)^\sigma y^\sigma d_1(z) - d_1(x)^\sigma y^\sigma k_1(z).\end{aligned}$$

[5, Corollary 3.6] gives

$$p_{ij} = [a_0, x_i]yk_0(z_j) + [b_0, x_i]yd_0(z_j) + k_0(x_i)yd_0(z_j) + d_0(x_i)yk_0(z_j) = 0, \tag{3.6}$$

$$\begin{aligned}q_{ij} =&a_1x_i^\sigma yk_1(z_j) - x_ia_1yk_1(z_j) - b_1x_i^\sigma yd_1(z_j) + x_ib_1yd_1(z_j)\\ &+ k_1(x_i)^\sigma yd_1(z_j) - d_1(x_i)^\sigma yk_1(z_j) = 0.\end{aligned} \tag{3.7}$$

for all $x_i \in A_i$, $y \in A$, $z_j \in A_j$, $i, j \in \{0, 1\}$. Therefore

$$\begin{aligned}p_{00} + p_{01} + p_{10} + p_{11} =&[a_0, x]yk_0(z) + [b_0, x]yd_0(z) + k_0(x)yd_0(z)\\ &+ d_0(x)yk_0(z) = 0,\end{aligned} \tag{3.8}$$

$$\begin{aligned}q_{00} + q_{01} + q_{10} + q_{11} =&a_1x^\sigma yk_1(z) - xa_1yk_1(z) - b_1x^\sigma yd_1(z) + xb_1yd_1(z)\\ &+ k_1(x)^\sigma yd_1(z) - d_1(x)^\sigma yk_1(z) = 0.\end{aligned} \tag{3.9}$$

According to (3.8) and Lemma 3.3 we see that either (i) or (ii) or (iii) is true.

By (3.9) we get

$$\begin{aligned}&[a_1, x_0]yk_1(z) - [b_1, x_0]yd_1(z) - k_1(x_0)yd_1(z)\\ &\qquad + d_1(x_0)yk_1(z) = 0 \text{ for all } x_0 \in A_0, y, z \in A,\end{aligned}$$

$$-[a_1,x_1]_s y k_1(z)+[b_1,x_1]_s y d_1(z)+k_1(x_1)yd_1(z) \\ -d_1(x_1)yk_1(z)=0 \text{ for all } x_1\in A_1, y,z\in A.$$

Combining the identities above we give

$$[a_1,x]_s y k_1(z)-[b_1,x]_s y d_1(z)-k_1(x)yd_1(z)+d_1(x)yk_1(z)=0 \text{ for all } x,y,z\in A.$$

By Lemma 3.4 we have that either (i) or (ii) or (iii) is true. Similarly, using the same way to $l_1(xy)$ we have

$$[a_0,x]yk_1(z)+[b_0,x]yd_1(z)+k_0(x)yd_1(z)+d_0(x)yk_1(z)=0,$$

$$a_1xyk_0(z)-x^\sigma a_1yk_0(z)+b_1xyd_0(z) \\ -x^\sigma b_1yd_0(z)+k_1(x)yd_0(z)+d_1(x)yk_0(z)=0 \quad (3.10)$$

and either (i) or (ii) or (iii) is true. □

In particular, taking $a=b=0$ in Theorem 3.5 we obtain

3.6. Corollary. ([5, Theorem 4.1]) *Let A be a prime associative superalgebra and let $d=d_0+d_1$ and $k=k_0+k_1$ be nonzero superderivations on A. Then dk is a superderivation if and only if $d_0=k_0=0$ and $k_1=\lambda_0 d_1$ for some nonzero $\lambda_0\in C_0$.*

Proof. We assume that both d_0 and k_0 are nonzero. Since d and k are nonzero superderivations and dk is also a superderivation of A, then there exists $0\neq\mu\in C_0$ such that $k_0(x)=\mu d_0(x)$ by Theorem 3.5. We have $2\mu d_0(x)yd_0(x)=0$ by taking $z=x$ in (3.8), that is, $d_0(x)Ad_0(x)=0$. Since A is a semiprime algebra, then $d_0(x)=0$. But it contradicts $d_0\neq 0$. We set $d_0=0$. Then $d_1\neq 0$. When $k_1\neq 0$. There exists $0\neq\lambda_0\in C_0$ such that $k_1(x)=\lambda_0 d_1(x)$ and $k_0(x)=d_0(x)=0$ by Theorem 3.5. When $k_1=0$ and $k_0\neq 0$, we have $d_1(x)=0$ by (3.10). It contradicts that d is a nonzero superderivation. So $d_0=k_0=0$ and $k_1=\lambda_0 d_1$ for some nonzero $\lambda_0\in C_0$ when dk is a superderivation. It is easy to prove that dk is a superderivation when $d_0=k_0=0$ and $k_1=\lambda_0 d_1$ for some nonzero $\lambda_0\in C_0$ □

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