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Improved ratio-type estimators of finite population variance using quartiles

Ramkrishna S. Solanki*† Housila P. Singh‡ and Surya K. Pal§

Abstract

In this paper we have proposed some ratio-type estimators of finite population variance using known values of parameters related to an auxiliary variable such as quartiles with their properties in simple random sampling. The suggested estimators have been compared with the usual unbiased and ratio estimators and the estimators due to [2], [12, 13, 14] and [3]. An empirical study is also carried out to judge the merits of the proposed estimator over other existing estimators of population variance using natural data set.

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1. Introduction

Estimating the finite population variance has great significance in various fields such as industry, agriculture, medical and biological sciences where we come across the populations which are likely to be skewed. Variation is present everywhere in our day to day life. It is law of nature that no two things or individuals are exactly alike. For instance, a physician needs a full understanding of variation in the degree of human blood pressure, body temperature and pulse rate for adequate prescription. A manufacture needs constant knowledge of the level of variation in people's reaction to his product to be able to known whether to reduce or increase his price, or improve the quality of his product. An agriculturist needs an adequate understanding of variations in climate factors especially from place to place (or time to time) to be able to plan on when, how and where

*Department of Mathematics and Statistics, College of Agriculture, Balaghat, JNKVV, Jabalpur, M.P., India
Email: ramkssolanki@gmail.com
†Corresponding Author.
‡S.S. in Statistics, Vikram University, Ujjain, India, Email: hpsujn@gmail.com
§S.S. in Statistics, Vikram University, Ujjain, India, Email: suryakantpal6676@gmail.com

to plant his crop. In manufacturing industries and pharmaceutical laboratories some of times researchers are more interested about the variation of their products or yields. Many more situations can be encountered in practice where the estimation of population variance of the study variable assumes importance. For these reasons various authors have paid their attention towards the estimation of population variance. In sample surveys, auxiliary information on the finite population under study is quite often available from previous experience, census or administrative databases. The sampling literature describes a wide variety of techniques for using auxiliary information to improve the sampling design and/or obtain more efficient estimators of finite population variance. It is well known that when the auxiliary information is to be used at the estimation stage, the ratio method of estimation is extensively employed. The ratio estimation method has been extensively used because of its intuitive appeal, computational simplicity and applicability to a general design. Perhaps, this is why many researchers have directed their efforts toward to get more efficient ratio-type estimators of the population variance by modifying the structure of existing estimators. Such as, [2], [5], [6, 7], [8] and [11] have suggested some modified estimators of population variance using known values of coefficient of variation, coefficient of kurtosis, coefficient of skewness of an auxiliary variable together with their biases and mean squared errors. We have known that the value of quartiles and their functions are unaffected by the extreme values or the presence of outliers in the population values. For this reason, [3] and [12, 13, 14] have considered the problem of estimating the population variance of the study variable using information on variance, quartiles, inter-quartile range, semi-quartile range and semi-quartile average of an auxiliary variable. In this paper our main goal is to estimate the unknown population variance of the study variable by improving the estimators suggested previously using same information on an auxiliary variable such as quartiles, inter-quartile range, semi-quartile range, semi-quartile average etc. The remaining part of the paper is organized as follows: The Section 2 introduced the notations and some existing estimators of population variance in brief. In Section 3, the ratio-type estimator of population variance is suggested and the expressions of their asymptotic biases and the mean squared errors are obtained. In addition, some members of suggested ratio-type estimators are also generated with their properties. The Section 4 is addressed the problem of efficiency comparisons of proposed ratio-type estimators with the usual unbiased estimator and the estimator due to [1], while Section 5 is focused on empirical study of proposed ratio-type estimators for the real data set. We conclude with a brief discussion in Section 6.

2. Notations and literature review

Much literature has been produced on sampling from finite populations to address the issue of the efficient estimation of the variance of a survey variable when auxiliary variables are available. Our analysis refers to simple random sampling without replacement (SRSWOR) and considers, for brevity, the case when only a single auxiliary variable is used. Let $U = (U_1, U_2, ..., U_N)$ be finite population of size N and (y, x) are (study, auxiliary) variables taking values (y_i, x_i) respectively for the i^{th} unit U_i of the finite population U. Our quest is to estimate the unknown population variance $S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} \left(y_i - \bar{Y}\right)^2$ of study variable y, where $\bar{Y} = N^{-1} \sum_{i=1}^{N} y_i$ is the population mean of y. Let a simple random sample (SRS) of size n be drawn without replacement (WOR) from the finite population U. The usual unbiased estimator of finite population variance S_y^2 is defined as

$$(2.1) \qquad s_y^2 = t_0 = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

where $\bar{y} = n^{-1} \sum_{i=1}^{n} y_i$. [1] has suggested the usual ratio estimator of S_y^2 as

$$(2.2) \qquad t_R = t_1 = s_y^2 \left(\frac{S_x^2}{s_x^2} \right)$$

where $S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} \left(x_i - \bar{X}\right)^2$, $\bar{X} = N^{-1} \sum_{i=1}^{N} x_i$, $s_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ and $\bar{x} = n^{-1} \sum_{i=1}^{n} x_i$. Motivated by [10], [15] and [9], [2] have proposed following ratio-type estimators of the population variance as

$$(2.3) \qquad t_2 = s_y^2 \left(\frac{S_x^2 - C_x}{s_x^2 - C_x} \right)$$

$$(2.4) \qquad t_3 = s_y^2 \left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)} \right)$$

$$(2.5) \qquad t_4 = s_y^2 \left(\frac{\beta_2(x) S_x^2 - C_x}{\beta_2(x) s_x^2 - C_x} \right)$$

$$(2.6) \qquad t_5 = s_y^2 \left(\frac{C_x S_x^2 - \beta_2(x)}{C_x s_x^2 - \beta_2(x)} \right)$$

where $C_x = \left(S_x / \bar{X}\right)$ and $\beta_2(x)$ are the known coefficients of variation and kurtosis of the auxiliary variable x respectively. Using the known value of population median Q_2 of the auxiliary variable x [12]have suggested the ratio-type estimator of population variance S_y^2 as

$$(2.7) \qquad t_6 = s_y^2 \left(\frac{S_x^2 + Q_2}{s_x^2 + Q_2} \right)$$

[13] have proposed the modified ratio-type estimators of population variance S_y^2 of the study variable y using the known quartiles and their functions of the auxiliary variable x as

$$(2.8) \qquad t_7 = s_y^2 \left(\frac{S_x^2 + Q_1}{s_x^2 + Q_1} \right)$$

$$(2.9) \qquad t_8 = s_y^2 \left(\frac{S_x^2 + Q_3}{s_x^2 + Q_3} \right)$$

$$(2.10) \qquad t_9 = s_y^2 \left(\frac{S_x^2 + Q_r}{s_x^2 + Q_r} \right)$$

$$(2.11) \qquad t_{10} = s_y^2 \left(\frac{S_x^2 + Q_d}{s_x^2 + Q_d} \right)$$

$$(2.12) \qquad t_{11} = s_y^2 \left(\frac{S_x^2 + Q_a}{s_x^2 + Q_a} \right)$$

where Q_i is the i^{th} quartile $(i = 1, 3)$, $Q_r = (Q_3 - Q_1)$ (inter-quartile range) $Q_d = \left(\frac{Q_3 - Q_1}{2}\right)$ (semi-quartile range) and $Q_a = \left(\frac{Q_3 + Q_1}{2}\right)$ (semi-quartile average). Taking motivation form [2] and [12]; [14] have suggested the ratio-type estimators of population variance S_y^2 using known values of coefficient of variation C_x and population median Q_2 of an auxiliary variable x as

$$(2.13) \qquad t_{12} = s_y^2 \left(\frac{C_x S_x^2 + Q_2}{C_x s_x^2 + Q_2} \right)$$

Recently [3] have proposed another ratio-type estimator of population variance S_y^2 using known values of coefficient of correlation ρ between the variables (y, x) and population quartile Q_3 of an auxiliary variable x as

$$(2.14) \qquad t_{13} = s_y^2 \left(\frac{\rho S_x^2 + Q_3}{\rho s_x^2 + Q_3} \right)$$

To the first degree of approximation the biases and mean squared errors ($MSEs$) of the estimators t_j, $(j = 0, 1, 2, ..., 13)$ are respectively given as

(2.15) $\quad Bias\,(t_j) = \Phi\tau_j\,(\tau_j - c)$

(2.16) $\quad MSE\,(t_j) = \gamma\,[\lambda^*_{40} + \tau_j\lambda^*_{04}\,(\tau_j - 2c)]$

where $\Phi = \gamma\lambda^*_{04}$, $\gamma = n^{-1}S^4_y, c = \left(\lambda^*_{22}\lambda^{*-1}_{04}\right)$, $\tau_0 = 0$, $\tau_1 = 1$, $\tau_2 = S^2_x\left(S^2_x - C_x\right)^{-1}$, $\tau_3 = S^2_x\left(S^2_x - \beta_2\,(x)\right)^{-1}$, $\tau_4 = \beta_2\,(x)\,S^2_x\left(\beta_2\,(x)\,S^2_x - C_x\right)^{-1}$, $\tau_5 = C_xS^2_x\left(C_xS^2_x - \beta_2\,(x)\right)^{-1}$, $\tau_6 = S^2_x\left(S^2_x + Q_2\right)^{-1}$, $\tau_7 = S^2_x\left(S^2_x + Q_1\right)^{-1}$, $\tau_8 = S^2_x\left(S^2_x + Q_3\right)^{-1}$, $\tau_9 = S^2_x\left(S^2_x + Q_r\right)^{-1}$, $\tau_{10} = S^2_x\left(S^2_x + Q_d\right)^{-1}$, $\tau_{11} = S^2_x\left(S^2_x + Q_a\right)^{-1}$, $\tau_{12} = C_xS^2_x\left(C_xS^2_x + Q_2\right)^{-1}$, $\tau_{13} = \rho S^2_x\left(\rho S^2_x + Q_3\right)^{-1}$, $\lambda^*_{rs} = (\lambda_{rs} - 1)$, $\lambda_{rs} = \mu_{rs}\left(\mu^{s/2}_{02}\mu^{r/2}_{20}\right)^{-1}$, $\mu_{rs} = N^{-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^r\left(x_i - \bar{X}\right)^s$ (r, s being non negative integers). It is observed that the estimators $(t_6, t_7, ..., t_{13})$ due to [12, 13, 14] and [3] have used the quartiles and their functions such as inter-quartile range Q_r , semi-quartile range Q_d and semi-quartile average Q_a and in additive form to sample and population variances s^2_x and S^2_x respectively of the auxiliary variable x. It is to be noted that the unit of the quartiles and their function as given above is of original variable x, while the unit of S^2_x and s^2_x are in the square of the unit of the original variable x. These lead authors to develop a more justified ratio-type estimators of the population variance S^2_y of the study variable y using known values of parameters related to the auxiliary variable x and study their properties in simple random sampling.

3. The proposed ratio-type estimator

We propose following ratio-type estimators of population variance S^2_y in simple random sampling as

(3.1) $$T = s^2_y\left(\frac{\delta S^2_x + \alpha L^2}{\delta s^2_x + \alpha L^2}\right)$$

where $\left(\delta S^2_x + \alpha L^2\right) > 0, \left(\delta s^2_x + \alpha L^2\right) > 0$ and (δ, L) are either real constants or function of known parameters of an auxiliary variable x with $0 \leq \alpha \leq 1$. To obtain the bias and MSE of the proposed ratio-type estimator T, we write $s^2_y = S^2_y\,(1 + e_0)$ and $s^2_x = S^2_x\,(1 + e_1)$ such that $E\,(e_0) = E\,(e_1) = 0$ and to the first degree of approximation (ignoring finite population correction (f.p.c.) term), we have $E\left(e^2_0\right) = n^{-1}\lambda^*_{40}$, $E\left(e^2_1\right) = n^{-1}\lambda^*_{04}$, $E\,(e_0e_1) = n^{-1}\lambda^*_{22}$. Now expressing (3.1) in terms of e's, we have

(3.2) $$T = S^2_y\,(1 + e_0)\left[\frac{\delta S^2_x + \alpha L^2}{\delta S^2_x\,(1 + e_1) + \alpha L^2}\right] = S^2_y\,(1 + e_0)\,(1 + \tau^* e_1)^{-1}$$

where $\tau^* = \delta S^2_x\left(\delta S^2_x + \alpha L^2\right)^{-1}$. We assume that $|\tau^* e_1| < 1$ so that $(1 + \tau^* e_1)^{-1}$ is expendable. Expanding the right hand side of (3.2) and multiplying out, we have

$T = S^2_y\,(1 + e_0)\left(1 - \tau^* e_1 + \tau^{*2}e^2_1 - ...\right)$

$= S^2_y\left(1 + e_0 - \tau^* e_1 - \tau^* e_0e_1 + \tau^{*2}e^2_1 + \tau^{*2}e_0e^2_1 - ...\right)$

Neglecting terms of e's having power greater than the two, we have

$T \cong S^2_y\left(1 + e_0 - \tau^* e_1 - \tau^* e_0e_1 + \tau^{*2}e^2_1\right)$

or

(3.3) $$\left(T - S^2_y\right) \cong S^2_y\left(e_0 - \tau^* e_1 - \tau^* e_0e_1 + \tau^{*2}e^2_1\right)$$

Taking expectation of both sides of (3.3),we get the bias of the estimator $T$to the first degree of approximation as

(3.4) $$Bias\,(T) = \Phi\tau^*\,(\tau^* - c)$$

Squaring both sides of (3.3) and neglecting terms of e's having power greater than two, we have

$$(3.5)\qquad \left(T - S_y^2\right)^2 \cong S_y^4\left(e_0^2 + \tau^{*2}e_1^2 - 2\tau^* e_0 e_1\right)$$

Taking expectation of both sides of (3.5), we get the MSE of the estimator T to the first degree of approximation as

$$(3.6)\qquad MSE\,(T) = \gamma\left[\lambda_{40}^* + \tau^*\lambda_{04}^*\left(\tau^* - 2c\right)\right]$$

Below we have identified some members of proposed ratio type estimator T for different choices of (δ, L).

(i) The estimator based on coefficient of variation C_x and quartile Q_1:

If we set $(\delta, L) = (C_x, Q_1)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.7)\qquad T_1 = s_y^2\left(\frac{C_x S_x^2 + \alpha Q_1^2}{C_x s_x^2 + \alpha Q_1^2}\right)$$

(ii) The estimator based on coefficient of kurtosis $\beta_2(x)$ and median Q_2:

If we set $(\delta, L) = (\beta_2(x), Q_2)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.8)\qquad T_2 = s_y^2\left(\frac{\beta_2(x) S_x^2 + \alpha Q_2^2}{\beta_2(x) s_x^2 + \alpha Q_2^2}\right)$$

(iii) The estimator based on population mean $\bar{X}$ and quartile Q_3:

If we set $(\delta, L) = \left(\bar{X}, Q_3\right)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.9)\qquad T_3 = s_y^2\left(\frac{\bar{X} S_x^2 + \alpha Q_3^2}{\bar{X} s_x^2 + \alpha Q_3^2}\right)$$

(iv) The estimator based on coefficient of kurtosis $\beta_2(x)$ and inter-quartile range Q_r:

If we set $(\delta, L) = (\beta_2(x), Q_r)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.10)\qquad T_4 = s_y^2\left(\frac{\beta_2(x) S_x^2 + \alpha Q_r^2}{\beta_2(x) s_x^2 + \alpha Q_r^2}\right)$$

(v) The estimator based on correlation coefficient ρ and semi-quartile range Q_d:

If we set $(\delta, L) = (\rho, Q_d)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.11)\qquad T_5 = s_y^2\left(\frac{\rho S_x^2 + \alpha Q_d^2}{\rho s_x^2 + \alpha Q_d^2}\right)$$

(vi) The estimator based on correlation coefficient ρ and semi-quartile average Q_a:

If we set $(\delta, L) = (\rho, Q_a)$ in (3.1), we get the estimator of S_y^2 as,

$$(3.12)\qquad T_6 = s_y^2\left(\frac{\rho S_x^2 + \alpha Q_a^2}{\rho s_x^2 + \alpha Q_a^2}\right)$$

Similarly one can identify many other estimators from the proposed ratio-type estimator T for different combinations of (δ, L). To the first degree of approximation the biases and mean squared errors $(MSEs)$ of the estimators $T_k, (k = 1, 2, ..., 6)$ are respectively given by

$$(3.13)\qquad Bias\,(T_k) = \Phi\tau_k^*\left(\tau_k^* - c\right)$$

$$(3.14)\qquad MSE\,(T_k) = \gamma\left[\lambda_{40}^* + \tau_k^*\lambda_{04}^*\left(\tau_k^* - 2c\right)\right]$$

where $\tau_1^* = C_x S_x^2\left(C_x S_x^2 + \alpha Q_1^2\right)^{-1}$, $\tau_2^* = \beta_2(x) S_x^2\left(\beta_2(x) S_x^2 + \alpha Q_2^2\right)^{-1}$, $\tau_3^* = \bar{X} S_x^2\left(\bar{X} S_x^2 + \alpha Q_3^2\right)^{-1}$, $\tau_4^* = \beta_2(x) S_x^2\left(\beta_2(x) S_x^2 + \alpha Q_r^2\right)^{-1}$, $\tau_5^* = \rho S_x^2\left(\rho S_x^2 + \alpha Q_d^2\right)^{-1}$, $\tau_6^* = \rho S_x^2\left(\rho S_x^2 + \alpha Q_a^2\right)^{-1}$.

Table 1. The parameters of population data set

N	80	C_y	0.3542	Q_1	5.1500
n	20	S_x	8.4563	Q_2	10.300
$\bar{Y}$	51.8264	C_x	0.7507	Q_3	16.975
$\bar{X}$	11.2646	λ_{04}	2.8664	Q_r	11.825
ρ	0.9413	λ_{40}	2.2667	Q_d	5.9125
S_y	18.3569	λ_{22}	2.2209	Q_a	11.0625

Table 2. $PREs$ of estimators $t_j, (j = 0, 1, ..., 13)$ with respect to s_y^2

Percent relative efficiency (PRE)						
(t_0, s_y^2)	(t_1, s_y^2)	(t_2, s_y^2)	(t_3, s_y^2)	(t_4, s_y^2)	(t_5, s_y^2)	(t_6, s_y^2)
100.00	183.23	179.62	169.24	181.98	164.49	226.87
Percent relative efficiency (PRE)						
(t_7, s_y^2)	(t_8, s_y^2)	(t_9, s_y^2)	(t_{10}, s_y^2)	(t_{11}, s_y^2)	(t_{12}, s_y^2)	(t_{13}, s_y^2)
206.64	247.25	232.13	209.86	229.54	238.17	249.84

4. The theoretical evaluation

We have made some theoretical conditions under which the ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ which are members of proposed ratio-type estimator T are more efficient than the other existing estimators $t_j, (j = 0, 1, ..., 13)$ which are due to [1], [2], [12, 13, 14] and [3] respectively. From (2.16) and (3.14), we have

$MSE(T_k) < MSE(T_j)$ if $\tau_k^*(\tau_k^* - 2c) < \tau_j(\tau_j - 2c)$

i.e. if either,

$\tau_k^* < \tau_j$ and $c < \left(\frac{\tau_k^* + \tau_j}{2}\right)$

or,

$\tau_k^* > \tau_j$ and $c > \left(\frac{\tau_k^* + \tau_j}{2}\right)$

or equivalently ,

$\min.[\tau_j, (2c - \tau_j)] \leq \tau_k^* \leq \max.[\tau_j, (2c - \tau_j)], (j = 0, 1, ..., 13; k = 1, 2, ..., 6).$

5. Emprical study

The performance of the ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ which are members of the suggested ratio-type estimator T are evaluated against the usual unbiased estimator s_y^2 and the estimators $t_j, (j = 1, 2, ..., 13)$ which are due to [1], [2], [12, 13, 14] and [3] respectively. for the population data set [Source: [4]] summarized in Table 1. We have computed the percent relative efficiencies ($PREs$) of the estimators $t_j, (j = 1, 2, ..., 13)$ and the suggested ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ with respect to the usual unbiased estimator $t_0 = s_y^2$ in certain range of $\alpha \in (0.0, 1.0)$ by using following formulae respectively as

$$(5.1) \qquad PRE\left(t_j, s_y^2\right) = \frac{MSE\left(s_y^2\right)}{MSE\left(t_j\right)} \times 100 = \frac{\lambda_{40}^*}{[\lambda_{40}^* + \tau_j \lambda_{04}^*(\tau_j - 2c)]} \times 100$$

$$(5.2) \qquad PRE\left(T_k, s_y^2\right) = \frac{MSE\left(s_y^2\right)}{MSE\left(T_k\right)} \times 100 = \frac{\lambda_{40}^*}{[\lambda_{40}^* + \tau_k^* \lambda_{04}^*(\tau_k^* - 2c)]} \times 100$$

and finding are summarized in Tables 2 and 3. It is observed from Tables 2 and 3 that all the ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ which are members of proposed ratio-type estimator T performed better than the usual unbiased estimator s_y^2, usual ratio estimator

Table 3. $PREs$ of estimators $T_k, (k = 1, 2, ..., 6)$ with respect to s_y^2

	Percent relative efficiency (PRE)					
α	(T_1, s_y^2)	(T_2, s_y^2)	(T_3, s_y^2)	(T_4, s_y^2)	(T_5, s_y^2)	(T_6, s_y^2)
0.0	183.23	183.23	183.23	183.23	183.23	183.23
0.1	199.59	200.34	195.20	205.48	200.39	235.94
0.2	214.59	215.93	206.50	224.91	216.03	264.03
0.3	227.93	229.68	217.01	240.89	229.81	270.58
0.4	239.41	241.38	226.62	253.18	241.52	264.67
0.5	248.96	250.94	235.28	261.89	251.09	253.47
0.6	256.58	258.41	242.92	267.36	258.55	240.92
0.7	262.37	263.91	249.54	270.08	264.02	228.80
0.8	266.47	267.63	255.15	270.58	267.71	217.77
0.9	269.08	269.79	259.79	269.35	269.83	207.99
1.0	270.38	270.61	263.50	266.84	270.61	199.40

t_1 due to [1] and the estimators $t_j, (j = 2, 3, 4, 5)$ due to [2] for all $\alpha \in (0.0, 1.0)$. However all the ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ are more efficient than the estimators $t_j, (j = 6, 7, ..., 12)$ due to [12, 13, 14] and [3] for a specific value of α. The estimators T_2 and T_5 which utilize the information on $(\beta_2(x), Q_2)$ and (ρ, Q_d) respectively are the best in the sense of having largest percent relative efficiency among all the estimators discussed here for $\alpha = 1$.

6. Conclusion

In this paper we have suggested some ratio-type estimators of population variance S_y^2 of the study variable y using known parameters of an auxiliary variable such as coefficient of variation, coefficient of kurtosis, correlation coefficient and quartiles etc. The bias and mean squared error formulae of the proposed ratio-type estimators are obtained and compared with that of the usual unbiased estimator, traditional ratio estimator and the estimators due to [2], [12, 13, 14] and [3]. We have also assessed the performance of the proposed estimators for known natural population data set and found that the performances of the proposed estimators are better than the other existing estimator for certain cases.

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