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ANALYSIS OF TRIANGULAR CONTINGENCY TABLES

Serpil Aktaş* and Tülay Saraçbaşı*

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Abstract

This paper compares two methods involving the uniform association model and the quasi-independence model. These models can be described in terms of the association parameters for the analysis of triangular contingency tables having ordered categories. A simulation study based on 30,000 random triangular tables was performed for this comparison. Proportions of the rejected and accepted hypothesis under these models were obtained. From the results of the simulation study, the behaviour of the association parameters was discussed with respect to their coefficient of variations. The homogeneity of the coefficients of variance was also tested.

Keywords: Triangular contingency tables, Uniform association model, Quasi-independence model.

1. Introduction

Triangular contingency tables are a special class of incomplete contingency table which contain structural zeros in one or more cells above or below their main diagonals.

Triangular contingency tables were first analyzed in [7] by partitioning the table into a set of rectangular sub-tables, each of which can be analyzed in an elementary way. Bishop and Fienberg [4] illustrated this kind of table using the classical example of disability of stroke patients.

Altham [2], Mantel [11] and Bishop *et.al.* [5] also discussed the quasi-independence model. Goodman [10] introduced various tests of the quasi-independence model against an alternative hypothesis of positive or negative quasi-dependence. We consider $R \times R$ tables, where the row and the column categories are ordinal, numbered from 1 to R, and denote the probability that an observation falls in the ith row and jth column of the table.

Sarkar [12] defined four types of triangular contingency table using the following conditions: An upper-right (left) triangular (URT (ULT)) table is described by the condition that $\pi_{ij} = 0$ for $i > j$ (for $i + j > R + 1$), and a lower-left (right) triangular (LLT (LRT))

*Hacettepe University, Faculty of Science, Department of Statistics, 06532 Beytepe, Ankara.

table by the condition $\pi_{ij} = 0$ for $i < j$ (for $i + j < R + 1$). Each type can be transformed into the other by interchanging the row and column variables and/or reversing the category ordering. In this paper we make use of URT tables.

Due to the incompleteness of these tables, independence between the row and column variables is tested by the quasi-independence(QI) model first proposed in [7].

The QI model in a URT table is given as

$$\begin{aligned} \pi_{ij} &= \alpha_i \beta_j \quad (i \le j) \\ &= 0 \quad (i > j) \end{aligned} \tag{1}$$

where α_i and β_j are positive constants for $i = 1, \ldots, R$ and $j = 1, \ldots, R$.

For testing the QI model, Goodman [7], Bishop and Fienberg [4] used the usual chi-squared tests based on the Pearson and log-likelihood ratio statistics. On the other hand, Sarkar [12], considered testing the QI model in terms of ordinal associations based on concordant and discordant pairs of observations given by

$$C_\pi = 2 \sum_{i<k} \sum_{j<l} \pi_{ij}\, \pi_{kl}, \quad D_\pi = 2 \sum_{i>k} \sum_{j>l} \pi_{ij}\, \pi_{kl}.$$

The presence of the QI model can be described by $\eta_\pi = 0$, where

$$\begin{aligned} \eta_\pi &= C_\pi - D_\pi - C_{\pi^*} + D_{\pi^*} \\ &= 2 \sum_{i<k\le} \sum_{j<l} (\pi_{ij}\pi_{kl} - \pi_{il}\pi_{kj}). \end{aligned} \tag{2}$$

Hence the QI model can be interpreted by testing the null hypothesis $\mathrm{H}_0 : \eta_\pi = 0$ against the positive/negative likelihood ratio dependence $\mathrm{H}_A : \eta_\pi > 0$ ($\eta_\pi < 0$).

An appropriate asymptotic test for testing H_0 against H_A is provided by a right-tail test based on $z = \dfrac{\sqrt{n}\eta_{\widehat{\pi}}}{\widehat{\sigma}_{\widehat{\pi}}}$, whose asymptotic null distribution is $\mathrm{N}(0,1)$, where $\widehat{\sigma}_{\widehat{\pi}}$ is the maximum likelihood estimate of σ^2_π under H_0 is true [12].

For the analysis of a two-way cross classification table having ordered categories, Goodman[8] considered various kinds of association model. Suppose that both column and row variables of a two-dimensional table are ordinal, with row variable denoted by X and column variable denoted by Y. We assume that scores $\{u_i\}$ and $\{v_j\}$ are assigned to the rows and the columns, where $u_1 < u_2 < \cdots < u_R$, $v_1 < v_2 < \cdots < v_R$. The arithmetic means will be denoted by $\overline{u}$ and $\overline{v}$, respectively.

A simple loglinear model that uses the ordinal information, but which has only one more parameter than the usual independence model is given by

$$\log m_{ij} = \mu + \lambda_i^X + \lambda_j^Y + \beta^{XY} (u_i - \overline{u})(v_j - \overline{v}) \tag{3}$$

We will refer to this model as the *linear association model.* The parameter β in model (3) that describes the association between X and Y can be interpreted as the common value of the local log odds ratio. The independence model is the special case $\beta^{XY} = 0$. Goodman [8] suggested that model for the special case $\{u_i = i\}$, $\{v_j = j\}$, in which the local odds ratio θ_{ij} for adjacent rows i and $i + 1$, and adjacent columns j and $j + 1$, is uniformly $\exp(\beta)$. He referred to that special case as the *uniform association* (UA) model. A classic triangular table discussed by Bishop and Fienberg[4] given in Table 1 has been analyzed by several authors. In this table, the initial and final disability states of 121 stroke patients are graded on a five point scale A–E of increasing severity. None of the patients whose condition was worse than at admission were discharged.

Table 1. Initial and final ratings on disability of 121 stroke patients

	Final				
Initial	E	D	C	B	A
E	8	15	12	23	11
D	-	1	4	10	9
C	-	-	4	4	6
B	-	-	-	5	4
A	-	-	-	-	5

On analysing the data, the LR statistic is found to be 9.60 with 6 d.f. for the QI model. This value is significant at the level of 5%. For the uniform association model with integer scores, the Pearson statistic is 5.7438 based on 5 d.f. which is significant at the 1% level, and the maximum likelihood estimate of β is obtained as 0.2849 with estimated standard error of 0.1528. Here $\exp(\widehat{\beta}) = 1.329$ can be interpreted as the local odds ratio. When the method proposed in [12] is used, $\eta_{\widehat{\pi}}$ is found to be 0.0766 with estimated standard error 0.0345, and the z statistic is obtained as 2.2179. This value rejects $\mathrm{H}_0 : \eta_\pi = 0$ against $\mathrm{H}_A : \eta_\pi > 0$ at the level of 5%. This means that positive dependence occurs for the data.

2. Simulation Study

Although various methods have been discussed for analysing triangular tables, choosing the proper method is still of interest.

The method for triangular contingency tables should take into account the ordinal nature of the table. We perform a simulation study to compare the UA and QI methods, through association. For generating the random tables, we have used the bivariate normal distribution approach [9]. Samples drawn from a bivariate normal distribution with known correlation coefficient were transformed into contingency tables by making equal-interval frequency tables. After data discretization, simulation parameters were set as: sample size $n = 100, 250, 500, 1000$, correlation coefficients $\rho = 0.0; 0.2; 0.4$ and dimensions $R = 4, 5, 6, 7$, and 8.

To generate non correlated data, correlation coefficients were taken to be small. At the beginning of the simulation study, no statistically significant difference was found between $\rho = 0.0$ and $\rho = 0.2$; $\rho = 0.3$ and $\rho = 0.4$. Hence, ρ was taken to be 0.0, 0.2 and 0.4. After 500 replications in each combination, 30,000 URT tables were generated [1]. The UA and QI models were applied to these tables and the proportion of rejected to accepted hypothesis were found for each of the 500 replications. These are shown under the QI and UA models in Table 2 and Table 3.

The maximum likelihood estimates of β and η_π are summarized, together with their standard errors for each sample, in Table 4. Correlation coefficients between the two parameter estimates are presented in Table 5.

The Coefficient of Variation (CV) for the parameter estimates were also obtained to see the variation. The homogeneity of two coefficients of variation were tested by the likelihood ratio test [3,6] as shown in Table 6. The mean squared errors of both estimates are also given in Table 7. The UA model was assessed using the SAS statistical package release 6.12, using the GENMOD procedure to correct the degrees of freedom with respect to the cells containing structural zeros. Each row denotes the table combinations with respect to the sample size, correlation coefficient and the dimension.

Table 2. Proportions of accepted hypotheses under the QI model

n	ρ	R	%
100	0.0	4	79
100	0.0	5	74
100	0.0	6	79
100	0.0	7	81
100	0.0	8	87
100	0.2	4	77
100	0.2	5	76
100	0.2	6	82
100	0.2	7	83
100	0.2	8	91
100	0.4	4	79
100	0.4	5	82
100	0.4	6	82
100	0.4	7	81
100	0.4	8	91
250	0.0	4	53
250	0.0	5	50
250	0.0	6	58
250	0.0	7	63
250	0.0	8	70
250	0.2	4	51
250	0.2	5	53
250	0.2	6	58
250	0.2	7	60
250	0.2	8	71
250	0.4	4	52
250	0.4	5	51
250	0.4	6	56
250	0.4	7	64
250	0.4	8	73

n	ρ	R	%
500	0.0	4	24
500	0.0	5	20
500	0.0	6	23
500	0.0	7	29
500	0.0	8	30
500	0.2	4	25
500	0.2	5	18
500	0.2	6	22
500	0.2	7	22
500	0.2	8	34
500	0.4	4	22
500	0.4	5	16
500	0.4	6	19
500	0.4	7	30
500	0.4	8	52
1000	0.0	4	3
1000	0.0	5	2
1000	0.0	6	1
1000	0.0	7	1
1000	0.0	8	2
1000	0.2	4	3
1000	0.2	5	2
1000	0.2	6	1
1000	0.2	7	1
1000	0.2	8	2
1000	0.4	4	2
1000	0.4	5	1
1000	0.4	6	1
1000	0.4	7	1
1000	0.4	8	2

Table 3. Proportions of accepted hypotheses under the UA model

n	ρ	R	%
100	0.0	4	92
100	0.0	5	89
100	0.0	6	86
100	0.0	7	90
100	0.0	8	90
100	0.2	4	94
100	0.2	5	88
100	0.2	6	91
100	0.2	7	91
100	0.2	8	91
100	0.4	4	93
100	0.4	5	91
100	0.4	6	89
100	0.4	7	89
100	0.4	8	91
250	0.0	4	91
250	0.0	5	79
250	0.0	6	82
250	0.0	7	80
250	0.0	8	80
250	0.2	4	90
250	0.2	5	84
250	0.2	6	81
250	0.2	7	76
250	0.2	8	79
250	0.4	4	90
250	0.4	5	77
250	0.4	6	77
250	0.4	7	79
250	0.4	8	81

n	ρ	R	%
500	0.0	4	90
500	0.0	5	81
500	0.0	6	82
500	0.0	7	73
500	0.0	8	54
500	0.2	4	92
500	0.2	5	76
500	0.2	6	73
500	0.2	7	54
500	0.2	8	58
500	0.4	4	90
500	0.4	5	72
500	0.4	6	65
500	0.4	7	71
500	0.4	8	78
1000	0.0	4	85
1000	0.0	5	60
1000	0.0	6	55
1000	0.0	7	47
1000	0.0	8	42
1000	0.2	4	89
1000	0.2	5	66
1000	0.2	6	53
1000	0.2	7	21
1000	0.2	8	36
1000	0.4	4	89
1000	0.4	5	67
1000	0.4	6	38
1000	0.4	7	30
1000	0.4	8	30

$P > 0.05$

3. Conclusion

The QI model does not take into account the ordinal nature of the row and column variables. In order to avoid this problem we applied the UA model to the generated tables, taking integer scores and interpreting the QI model in terms of the ordinal association. Table 2 and Table 3 present the proportions of accepted hypothesis under the QI and UA models for the 500 replications. It can be seen that, when the sample size increases the proportion of accepted hypothesis decreases for both models.

Table 4. Estimated parameters and their standard errors

No.	n	ρ	R	$\widehat{\beta}$	SE of $\widehat{\beta}$	$\eta_{\widehat{\pi}}$	SE of $\eta_{\widehat{\pi}}$	No.	n	ρ	R	$\widehat{\beta}$	SE of $\widehat{\beta}$	$\eta_{\widehat{\pi}}$	SE of $\eta_{\widehat{\pi}}$
1	100	0.0	4	0.5004	0.0160	0.0621	0.0018	31	500	0.0	4	0.5171	0.0064	0.0488	0.00057
2	100	0.0	5	0.3409	0.0120	0.0714	0.0022	32	500	0.0	5	0.4066	0.0052	0.0696	0.00078
3	100	0.0	6	0.2418	0.0082	0.0725	0.0022	33	500	0.0	6	0.3058	0.0036	0.0745	0.00074
4	100	0.0	7	0.1810	0.0066	0.0739	0.0024	34	500	0.0	7	0.2230	0.0029	0.0755	0.00078
5	100	0.0	8	0.1268	0.0044	0.0676	0.0021	35	500	0.0	8	0.1677	0.0025	0.0731	0.00093
6	100	0.2	4	0.5272	0.0160	0.0654	0.0018	36	500	0.2	4	0.5218	0.0070	0.0482	0.00062
7	100	0.2	5	0.3213	0.0110	0.1399	0.0210	37	500	0.2	5	0.4127	0.0057	0.0692	0.0009
8	100	0.2	6	0.2456	0.0076	0.0729	0.0200	38	500	0.2	6	0.3129	0.0042	0.0730	0.00084
9	100	0.2	7	0.1809	0.0063	0.0722	0.0021	39	500	0.2	7	0.2262	0.0032	0.0757	0.00092
10	100	0.2	8	0.1286	0.0046	0.0680	0.0022	40	500	0.2	8	0.1692	0.0024	0.0738	0.00086
11	100	0.4	4	0.4959	0.0150	0.0634	0.0018	41	500	0.4	4	0.5404	0.0076	0.0497	0.00068
12	100	0.4	5	0.3426	0.0120	0.0690	0.0021	42	500	0.4	5	0.4291	0.0057	0.0713	0.0009
13	100	0.4	6	0.2362	0.0076	0.0700	0.0021	43	500	0.4	6	0.3034	0.0043	0.0730	0.00087
14	100	0.4	7	0.1713	0.0059	0.0233	0.0024	44	500	0.4	7	0.2254	0.0028	0.0754	0.00073
15	100	0.4	8	0.1282	0.0042	0.0676	0.0020	45	500	0.4	8	0.1705	0.0021	0.0740	0.00073
16	250	0.0	4	0.5184	0.0110	0.0555	0.0011	46	1000	0.0	4	0.5424	0.0052	0.0425	0.00042
17	250	0.0	5	0.3738	0.0077	0.0698	0.0013	47	1000	0.0	5	0.4342	0.0047	0.0671	0.00063
18	250	0.0	6	0.2768	0.0059	0.0737	0.0014	48	1000	0.0	6	0.3378	0.0031	0.0725	0.00053
19	250	0.0	7	0.1997	0.0040	0.0724	0.0012	49	1000	0.0	7	0.2557	0.0024	0.0761	0.00051
20	250	0.0	8	0.1493	0.0036	0.0698	0.0014	50	1000	0.0	8	0.1967	0.0019	0.0763	0.00054
21	250	0.2	4	0.5099	0.0110	0.0551	0.0011	51	1000	0.2	4	0.5529	0.0051	0.0438	0.00042
22	250	0.2	5	0.3728	0.0076	0.0696	0.0013	52	1000	0.2	5	0.4368	0.0043	0.0675	0.00057
23	250	0.2	6	0.2815	0.0055	0.0749	0.0013	53	1000	0.2	6	0.3337	0.0032	0.0713	0.00054
24	250	0.2	7	0.1992	0.0044	0.0727	0.0014	54	1000	0.2	7	0.2417	0.0028	0.0749	0.00068
25	250	0.2	8	0.1553	0.0035	0.0730	0.0014	55	1000	0.2	8	0.1927	0.0019	0.0713	0.00059
26	250	0.4	4	0.5179	0.0110	0.0555	0.0011	56	1000	0.4	4	0.5543	0.0047	0.0435	0.00037
27	250	0.4	5	0.3739	0.0082	0.0702	0.0014	57	1000	0.4	5	0.4428	0.0041	0.0699	0.00053
28	250	0.4	6	0.2753	0.0056	0.0742	0.0013	58	1000	0.4	6	0.3222	0.0038	0.0714	0.00069
29	250	0.4	7	0.2063	0.0043	0.0748	0.0013	59	1000	0.4	7	0.2536	0.0029	0.0763	0.00069
30	250	0.4	8	0.1517	0.0030	0.0726	0.0012	60	1000	0.4	8	0.1872	0.0021	0.0743	0.0062

The proportion of accepted hypotheses under the UA model is much higher than for the QI model. Hypotheses are not effected by the correlation coefficient and dimension. In Table 4, $\eta_{\widehat{\pi}}$ is seen to be closer to zero than $\widehat{\beta}$ under the assumption of no correlation. Although $\widehat{\beta}$ has been affected by the sample size and dimension, $\eta_{\widehat{\pi}}$ has not; note that $\widehat{\beta}$ increases as the dimension increases, but $\eta_{\widehat{\pi}}$ remains stable.

In Table 5, we investigate the correlation between the two estimates and it is clear that $\widehat{\beta}$ and $\eta_{\widehat{\pi}}$ are highly correlated and statistically significant with respect to their probabilities. In sample numbers 35 and 55, an incongruity from the expected is observed. It is thought that the random sampling may have caused these results.

From the results in Table 6, the homogeneity tests for each sample are almost non-significant, which means that there is no statistical difference between the CV of $\widehat{\beta}$ and that of $\eta_{\widehat{\pi}}$.

From the results of Table 7, we conclude that the minimum mean squared error is found for the QI model. As a result of the simulation study, we can say that $\eta_{\widehat{\pi}}$ gives better estimates under the assumption that the quasi-independence model holds true. Both estimates are highly correlated. Hence $\eta_{\hat{\pi}}$ can be considered as giving better estimates for the analysis of triangular tables. In further studies both estimates will be compared using different criteria.

Table 5. Pearson correlation coefficients between the two estimates

Sample No	r	Prob	Sample No	r	Prob
1	0.9530	0.00	31	0.8770	0.00
2	0.9500	0.00	32	0.1130	0.01
3	0.9360	0.00	33	0.8970	0.00
4	0.9390	0.00	34	0.8900	0.00
5	0.9330	0.00	35	-0.1490	0.00
6	0.9580	0.00	36	0.9050	0.00
7	0.4550	0.00	37	0.9300	0.00
8	0.9350	0.00	38	0.9260	0.00
9	0.9300	0.00	39	0.9010	0.00
10	0.9150	0.00	40	0.8960	0.00
11	0.9600	0.00	41	0.9050	0.00
12	0.9400	0.00	42	0.9190	0.00
13	0.9360	0.00	43	0.9290	0.00
14	0.2080	0.00	44	0.8910	0.00
15	0.9200	0.00	45	0.8980	0.00
16	0.9270	0.00	46	0.8410	0.00
17	0.9240	0.00	47	0.8960	0.00
18	0.9280	0.00	48	0.8570	0.00
19	0.9110	0.00	49	0.8590	0.00
20	0.9280	0.00	50	0.8600	0.00
21	0.9310	0.00	51	0.8140	0.00
22	0.9220	0.00	52	0.8770	0.00
23	0.9220	0.00	53	0.8640	0.00
24	0.9150	0.00	54	0.8750	0.00
25	0.9150	0.00	55	-0.0640	0.16
26	0.9410	0.00	56	0.7690	0.00
27	0.9300	0.00	57	0.8800	0.00
28	0.9250	0.00	58	0.9030	0.00
29	0.9110	0.00	59	0.8760	0.00
30	0.9270	0.00	60	0.8730	0.00

Table 6. Coefficients of Variation for $\widehat{\beta}$ and $\eta_{\widehat{\pi}}$ and the homogeneity test

Sample No	CV of $\widehat{\beta}$	CV of $\eta_{\widehat{\pi}}$	Z	Sample No	CV of $\widehat{\beta}$	CV of $\eta_{\widehat{\pi}}$	Z
1	3.1974	2.8985	-0.5841	31	1.2761	1.1680	-0.4341
2	3.5210	3.0812	-0.7575	32	1.2788	1.1207	-0.9859
3	3.3912	3.0345	-0.6367	33	1.1772	0.9933	-1.2765
4	3.6464	3.2476	-0.6381	34	1.3004	1.0331	-1.7051
5	3.4700	3.1065	-0.6209	35	1.4907	1.2722	-1.1544
6	3.0349	2.7523	-0.5950	36	1.3415	1.2448	-0.5558
7	3.4236	15.0107	2.5449*	37	1.3811	1.3006	-0.4439
8	0.0945	27.4348	1.8174**	38	1.1507	1.3423	-1.1414
9	3.4826	2.9086	3.4826*	39	1.2153	1.4146	-1.1161
10	3.5769	3.2353	-0.5540	40	1.1653	1.4184	-1.4410
11	3.0248	2.8391	-0.3834	41	1.3682	1.4064	-0.2027
12	3.5026	3.0435	-0.8022	42	1.2623	1.3284	-0.3787
13	3.2176	3.000	-0.4076	43	1.1918	1.4170	-1.2750
14	3.4442	10.3004	2.8240*	44	0.9682	1.2422	-1.8538
15	3.2761	2.9586	-0.5870	45	0.9865	1.2317	-1.6528
16	2.1219	1.9819	-0.4650	46	0.9882	0.9587	0.2319
17	2.0599	1.8625	-0.6914	47	0.9388	1.0824	-1.0794
18	2.1315	1.8996	-0.7818	48	0.7310	0.9177	-1.7361
19	2.0030	1.6574	2.0030*	49	0.6702	0.9386	-2.5402*
20	2.4112	2.0057	-1.2053	50	0.7077	0.9659	-2.3460
21	2.1573	1.9964	-0.5265	51	0.9589	0.9224	0.2973
22	2.0386	1.8678	-0.6012	52	0.9844	0.8444	-1.1716
23	1.9538	1.7356	-0.8214	53	0.7574	0.9589	-1.7958**
24	2.2088	1.9257	-0.9207	54	0.9078	1.1584	-1.8258**
25	2.2537	1.9178	-1.0766	55	0.8275	0.9859	-1.3329
26	2.1239	1.8018	-1.1291	56	0.8506	0.8479	0.0240
27	2.1931	1.9943	-0.6410	57	0.7582	0.9259	-1.5289
28	2.0341	1.7520	-1.0275	58	0.9964	1.1794	-1.4952
29	2.0843	1.7379	-1.2418	59	0.8257	1.1435	-2.4265*
30	1.9776	1.6529	-1.2381	60	0.8344	1.1218	-2.2116*

$^*P < 0.05$ $\quad$ $^{**}P < 0.10$

Table 7. Mean Squared Errors for $\widehat{\beta}$ and $\eta_{\widehat{\pi}}$

Sample No	MSE of $\widehat{\beta}$	MSE of $\eta_{\widehat{\pi}}$	Sample No	MSE of $\widehat{\beta}$	MSE of $\eta_{\widehat{\pi}}$
1	0.374968	0.005497	31	0.288275	0.002552
2	0.183961	0.007511	32	0.179305	0.005161
3	0.091849	0.007698	33	0.100029	0.005831
4	0.054451	0.008281	34	0.054093	0.006031
5	0.025952	0.006732	35	0.031298	0.005781
6	0.399906	0.005829	36	0.297219	0.002519
7	0.163074	0.021906	37	0.187153	0.005201
8	0.089326	0.007352	38	0.106927	0.005692
9	0.052391	0.007518	39	0.056521	0.006170
10	0.027264	0.006975	40	0.031585	0.005836
11	0.359125	0.005637	41	0.321211	0.002709
12	0.188951	0.007061	42	0.200871	0.005505
13	0.084586	0.007040	43	0.101697	0.005726
14	0.047112	0.003363	44	0.054838	0.005969
15	0.025167	0.006661	45	0.031300	0.005752
16	0.324791	0.003651	46	0.308443	0.001897
17	0.169587	0.005714	47	0.200111	0.004710
18	0.094307	0.006388	48	0.119148	0.005411
19	0.047849	0.006018	49	0.068421	0.005937
20	0.028718	0.005901	50	0.040491	0.005979
21	0.316045	0.003602	51	0.319547	0.002013
22	0.168035	0.005643	52	0.200246	0.004731
23	0.094511	0.006416	53	0.116698	0.005242
24	0.049470	0.006226	54	0.062507	0.005862
25	0.030388	0.006293	55	0.039071	0.005985
26	0.331705	0.003701	56	0.318998	0.001961
27	0.173888	0.005882	57	0.204865	0.004631
28	0.091685	0.006353	58	0.111306	0.005348
29	0.051706	0.006400	59	0.068617	0.006072
30	0.027708	0.006048	60	0.037404	0.005725

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