Lot Streaming Techniques

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Abstract- In batch manufacturing, the items are grouped into batches. These batches are treated as individual jobs and they are not divided. However, if batch sizes are large, it may be beneficial to split these batches into smaller transfer lots which is known as lot streaming. Lot streaming provides acceleration of the production by processing the items of the original batch in an overlapping fashion. In this paper, the lot streaming problem is defined, the problem characteristics and the notation are explained. Then the solution procedures in the literature for lot streaming problems are discussed respectively for 2-machine, 3-machine and, m-machine cases. For all types of 2 and 3-machine problems, there exist polynomial time algorithms. For certain m-machine problems, there also exist polynomial time algorithms.

Keywords Lot Streaming, Optimization, NP-Complete.

1. Introduction

Traditionally batch manufacturing has an important place in production scheduling. In batch manufacturing, items are grouped into several batches and each batch is treated as an individual job. In this case when a batch goes to a resource for an operation, all the items belonging to this batch have to wait until every item is processed. After the processing of the last item, the batch is ready for the next operation.

In general, if there are k items in a batch and each item has a processing time of p_i time units for each resource i, then this batch requires a processing time of $k \times p_i$ time units on each resource i. This means, on the average at each resource, an item waits $(k \times p_i)/2$ time units after being processed. If the batch size k is large then it may be beneficial to divide this batch into smaller sublots which can be transferred independently between resources.

The idea of dividing the batch into smaller transfer lots is the origin of lot streaming. This term was introduced by Reiter [9]:

"Lot Streaming is the process of splitting a process lot into sublots, and then scheduling those sublots in overlapping fashion, in order to accelerate the progress of an order in production." [6]

Today, synchronized manufacturing, which can be defined as "Any systematic way that attempts to move material quickly and smoothly through the various resources of the plant in concert with market demand" [4], gained much importance. In synchronized manufacturing, the process batch that is the quantity of a product considered for processing in a resource does not need to be equal to the transfer batch that is the quantity of units that are moved at the same time from one resource to the next [11]. In fact the idea of different transfer and process batches is same as what lot streaming does. For that reason, lot streaming has an important place in synchronized manufacturing systems such as Just In Time (JIT). By lot streaming, it is possible to accelerate the processing of the products at the same time reducing the work in process (WIP) inventory level and these are the aims of all manufacturing firms.

As an example, we can consider a batch with 500 items. If each item has a processing time of 2 time units on machine 1 ($p_1 = 2$) and 1 time unit on machine 2 ($p_2 = 1$), then according to the traditional batch manufacturing, the makespan will be 500x2 + 500x1 = 1500.

Next, if we consider to split this batch into two sublots with sizes 300 and 200, then it takes 300x2 = 600 time units for the items of the 1st sublot to be ready for machine 2. In the same manner, the items of the 2nd sublot will be ready for machine 2 after 300x2 + 200x2 = 1000 time units. As seen in Figure 1, dividing this batch into two sublots provides a makespan of 1200 time units which is smaller than the makespan under the original case. This small example gives an idea about how lot streaming accelerates the process of the items.

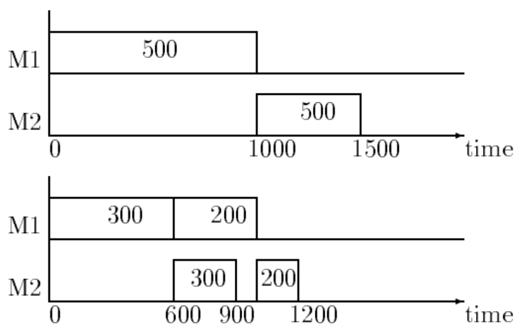

Fig. 1. Lot streaming effect on batch of 500 items on 2 machines

In the next section, the characteristics of the lot streaming problem, its notation and models are explained. In sections 3 and 4, the solution procedures for 2 and 3-machine cases are examined, respectively. In section 5, the case of having more than 3 machines is discussed, and finally in section 6, there is a brief overview of the ideas covered in this paper.

2. Lot Streaming Model and Notation

Although there is not a common notation in general, the notations used in different sources for the lot streaming problem is similar to each other. In this paper, the notation given in [3] is used.

In the lot streaming problem, there is an original job lot which has U identical items. These items are required to be processed by m machines. (Assuming that no machine is used for more than one operation) Each item has a processing time of p_i on machine i.

In most of the lot streaming problems, the number of sublots is a given parameter of the problem. Here, the number of sublots is denoted by n. If $n = 1$, this means that lot streaming is not allowed. In this case the makespan M will be:

$$M = U \times \sum_{i=1}^{m} p_i \qquad (1)$$

If $n = U$, then we are allowed to transfer each item one by one between the machines. If there is no setup time, for $n = U$, then the minimum makespan can be achieved and it is:

$$M = U \times \sum_{i=1}^{m-1} p_i + U \times p_m \qquad (2)$$

2.1. Sublot Sizes

Suppose that there are n sublots for each machine and also there are m different machines. As a result, in total there are $n \times m$ sublots in the model. These sublots are represented by L_{ij} where L_{ij} shows the size of the j^{th} sublot on machine i. Alternatively, x_{ij} shows the proportion of the size of the jth sublot on machine i over the total lot size U, in other words, $x_{ij} = L_{ij}/U$.

Sometimes, in lot streaming models, equal lot sizes are assumed. In such a case, all sublots have the same size and $L_{ij} = L \ \forall i, j$.

Another case is the model with consistent sublots. If the sublot sizes are same for every machine, then the sublots are called consistent and $L_{ij} = L_j$.

Finally, the most general case related to sublot size is the model with variable sublots. If all L_{ij} values are allowed to be different than each other, then it is said that variable lot sizes are allowed. Since equal and consistent sublots can be represented by using variable sublots, variable sublot case is dominant over equal and consistent sublot cases.

2.2. Discrete versus Continuous Models

The models can be divided into two classes as continuous or discrete case. In continuous case, sublot sizes can be real numbers, however in discrete case, sublots should have discrete number of items. If there is a solution procedure for finding optimal lot sizes for continuous case, it is possible to find an answer for discrete case by rounding off the results of the continuous case, but the optimality is not guaranteed.

2.3. No Idling versus Intermittent Idling

In some lot streaming problems, there is a constraint which requires that when a machine starts processing, it should process continuously all the items without being idle. This requirement is called "No Idling". If the machines are allowed to have idle time between the processes of two consecutive sublots, then this case is called "Intermittent Idling".

It is clear that for the 1st machine to have no idling restriction does not affect the makespan M. Also, it is known that: A lot streaming problem and its inverse are equivalent [2]. In other words, we can obtain the same optimal solution assuming that lots flow in the reverse order. By combining these two ideas, we can conclude that for 2-machine problem, no idling restriction does not affect the optimal makespan. If there are m machines in a model with no idling restriction and if variable sublots are allowed, it is possible to find optimal sublots by dividing m machines into m-1 adjacent pairs and solving the 2-machine problem for each pair separately [3].

2.4. Notation

To solve a lot streaming problem, we need information about certain characteristics of the problem. This information can be represented by the following way which is given in [3]:

m : Number of machines

E,C,V: Properties of sublot sizes: (*E* for Equal, *C* for Consistent, *V* for Variable sublot sizes)

II , NI: Idling condition (*II* for Intermitted Idling and *NI* for No Idling)

DV, CV: Discrete (*DV*) or Continuous (*CV*) case

Then according to the above information, each lot streaming problem can be represented in a form by using the notation:

m / V, C or *E / II* or *NI / CV* or *DV*

As an example, *2/C/NI/CV* represents a lot streaming problem which has two machines, consistent sublots, no idling restriction and it may have real numbered lot sizes.

2.5. Dominance

Related to the sublot sizes, variable sublot case is dominant over consistent sublot case which is dominant over equal sublot case. This means that a model with variable sublots should have shorter or equal makespan than the makespan of the same model with consistent or equal sublots.

It is clear that *II* (Intermittent Idling) dominates *NI* (No Idling) case and *CV* (Continuous) dominates over *DV* (Discrete) case.

According to these dominance relationships, the least restrictive model is *m/V/II/CV*.

3. Two-Machine Problems

In 2-machine problems, there are *n* sublots and transfer of sublots occurs between 1st and 2nd machines. As explained before, allowing intermittent idling does not improve the makespan. For that reason, optimal solution can be obtained by solving *2/C/NI* problem. In [2], this problem is solved and it is shown that if there are *n* sublots and the ratio of the processing times of machine 1 to machine 2, that is $p_2=p_1=q$, then for continuos version, the optimal solution for j^{th} sublot can be found by solving equation (3):

$$L_j = q \times L_{j-1} = q^{j-1} \times L_1 \quad (3)$$

In optimal solution of *2/C/NI* problem, there should not be idle time on machine 2 after it starts processing. In fact, the equation $L_j = q \times L_{j-1}$ is the condition for the lot sizes that provide no idling on machine 2. To understand this better, we can consider a 2/C/NI problem for which $U = 7$, $n = 3$, $p_1 = 2$ and $p_2 = 1$. Then according to the above formula, the optimal lot sizes will be $L_1 = 4$, $L_2 = 2$ and $L_3 = 1$. In Figure 2, it is seen that for these lot sizes there is no idle time on machine 2 after it starts processing.

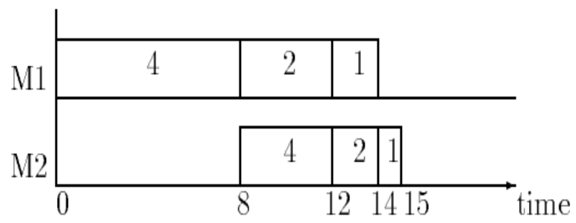

Fig. 2. An example for *2/C/NI* problem

A solution procedure exists also for *2/C/NI/DV* problem. In this procedure, initially an upper bound for the makespan *M* is determined. Then, by an iterative procedure, sublot sizes are determined. If during iterations, it is found that the optimal makespan is greater than the initially estimated *M* value, this *M* value is increased by some determined amount and the iterations are done again for the new *M* value. The interested reader is referred to [3] for more details.

For 2-machine problem, it is also possible to consider the *setup times* and *limited transporter capacity*. Baker [7], investigates 2-machine problem with setup time. He considers both attached and separable setup cases and defines the optimality condition.

Related to the 2-machine problem with limited transporter capacity, the solution for the problem 2/C/NI/CV with k transporters with cycle time *CT* is found again by the formula $L_j = q \times L_{j-1}$ where:

$$L_1 = \begin{cases} \dfrac{(CT/p_2) * (1-q)}{1-q^k} & \text{if } q \neq 1 \\ \dfrac{(CT/p_2)}{k} & \text{if } q = 1 \end{cases} \quad (4)$$

and

$$L_n = U - \sum_{i=1}^{n-1} L_j \quad (5)$$

The reader can find the derivation of equations (4) and (5) in [3]. In the same paper, the iterative solution procedures for *2/C/NI/DV* and *2/C/NI/CV* with limited transporter capacity are also given.

4. Three-Machine Problems

For 2-machine problem, it is enough to consider consistent sublot sizes to find the optimal lot sizes. However, in 3-machine problem, we may not have consistent sublots in the optimal solution. So, for 3-machine problem, minimum makespan *M* can be obtained by having variable sublots.

4.1. Three-Machine Problems with Consistent Sublots

For 3 or more machines, consistent sublot problem for continuous version can be solved by a linear programming (LP) model which is shown by Baker [5]. There are two LP models. The first model is constructed by defining the relationship between the completion times of sublots and its objective is minimizing the completion time of the last sublot on the last machine. This objective value is same as makespan value. In the second model, idle time relationship between the machines are defined and the objective is minimizing the total idle time that occurs on the last machine till the process of the last sublot.

Clearly, for discrete version of a consistent sublot problem for 3 or more machines, the solution can be found by putting a constraint in LP model which requires that sublot sizes are integer. In this case, an integer linear programming problem should be solved to find the optimal lot sizes.

4.2. Three-Machine Problems with with Variable Sublots and Intermittent Idling

3/V//II problems can be divided into two as: *3/V/II/CV* or *3/V/II/DV.* For *3/V/II/CV* problem, a solution procedure is given in [1]. In this solution procedure, the inequality

$$(p^2) - p_1 * p_3 \leq 0 \quad (6)$$

is checked. If inequality (6) holds, then in the optimal solution, sublot sizes have the below relationship shown by equation (7):

$$L_{j+1} \times (p_1 + p_2) = L_j \times (p_2 + p_3) \quad (7)$$

If inequality (6) does not hold, then optimal sublot sizes can be found by dividing the 3-machine problem into two subproblems. In this case optimal lot sizes between machines 1 and 2, machines 2 and 3 are found separately. These are also optimal lot sizes for the original *3-machine* problem.

In fact inequality (6) specifies the critical path for optimal sublots in the network for *3-machine flow shop problem.* What the above solution procedure does is that it determines the critical path property in the network of a 3-machine flow shop problem for the optimal lot sizes. According to this information, it finds the optimal lot sizes which minimize the critical path length. The interested reader is referred to [1].

For the problem *3/V/II/DV* again the inequality (6) is checked. If this inequality does not hold, it is possible to find a solution by dividing the machines into two adjacent pairs and finding optimal lot sizes separately as done in the continuous version. If inequality holds then there exists an iterative procedure to find optimal lot sizes which can be found in [3].

5. Problems for More Than Three Machines

When the number of machines is greater than 3 ($m > 3$), the problems become more difficult. As it is stated at the beginning, *m/V/NI* problem can be solved by applying 2-machine solution procedure to every adjacent air of machines. If the problem has consistent sublot requirements, then it can be solved by Baker's LP model which is described in section 4.1.

For m/V/II/CV problem, it is possible to formulate a mixed integer linear programming model [8]. By some modifications, this model can be used to find optimal solution for some other objective functions such as minimum mean flow time or minimum mean unit completion time.

Also we can consider the lot streaming problems for job shops. Job shop case is different than flow shop case in a way that the sublots can go to the same machine more than once for different operations. A solution procedure for lot streaming problem in job shop can be found in [10]. The solution procedure described in this paper uses the idea of solving a lot-sizing problem with a given sequence of sublots on the machines and a standard job shop scheduling problem with fixed sublot sizes.

6. Conclusion

In this paper, lot streaming is defined and some basic lot streaming problems are discussed with solution procedures that exist in the literature. In this respect, first, the problem characteristics and notation are described. Next, 2-machine, 3-machine and *m*-machine cases are explained. It is seen that there exist polynomial time solution algorithms for all 2-machine and 3-machine problems. There exist also polynomial time algorithms for *m/V/NI/DV* and *m/V/N I/CV*, because these problems can be solved by applying 2-machine procedures to adjacent pairs of machines. In addition, *m/C/II/CV* and *m/C/NI/CV* can be solved in polynomial time, because they have LP models.

The problems *m/C/NI/DV*, *m/C/II/DV* and *m/V/II/DV* have integer linear programming models but there is not a proof for NP-completeness [3]. For *m/V/II/CV* problem, there exists a mixed integer programming model [8], but it does not have a proof for NP-completeness, either.

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