THREE EQUAL ALTITUDE PROBLEM, ASTROLABE, APPLICATION OF A NEW COMPUTATION METHOD AND ITS THEORY

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ABSTRACT. — The method described in this dissertation was investigated and discussed at first in 1785 by Aboe and then in 1808 by Gauss and later in 1811 by Delamber under the name of «Three equal altitude star problem.»

The primitiveness of fine mechanics and drawback in the precision technique of the 19th century was limiting the astronomers of those days to get their best results in time and latitude determinations.

Especially by using «Chandler's Almucantar» and «Beck's Nadir device>> one could get highly 1 second of arc accuracy in latitude and 1/2 second of time accuracy in longitude observations. So that the above-mentioned method was used by travelers particularly on the geographical expeditions.

At the beginning of the 20th century measuring tools developed to such an extent that the instruments, like universal theodolites, zenith telescopes, astrolabes, etc., have been carefully designed and constructed. So that the observational precision became very high and especially one could obtain $m_{\varphi} = \pm 0.''15$ and $m_L = \pm 0.^s01 \sec \varphi$.

In order to establish astronomical points on the wide extended areas for high accurate triangulation chains the above-mentioned limits pertaining 10 precision were adequate, but very often difficulties were confronted in operations on the moun ainous regions because of transportation of the heavy observational equipment.

In eliminating this disadvantage, both the geodesist and the instrument makers, have decided to furnish measuring devices of considerable small size and light weight, such as Gautier, Tavistock, Wild, etc., theodolites.

In this investigation the writer made use of one of the above-written instruments and found the opportunity to make several observations and practiced and tested different methods, which are described in the original text.

The writer developed theoretically a new computation method and derived formulae for computing the time and latitude correctio is and in addition to that he worked out a mean error formula for determining the accuracy of his principles.

He compared separately his observation system with J. Ball and Th. Niethammer fundamentals in order to give an idea how safe the results of his measurements were and how convenient the usage of his new ocular attachment was and at last how fast was the observational operations which were carried out against above-mentioned classical methods.

The writer gave also a detailed summary in Part 2, regarding the computational results of the different methods. By the utilization of this table one can have an immediate and relative estimation between time, latitude and prism angle corrections and of their accuracy.

At the end of this paper the writer developed a special chronometer rate formula by using the rates which were obtained from star observations and from the signal receptions.

With regard to the longitude computations, he made use of two different chronometer rate values which were calculated on the base of time signal receptions and the formula mentioned above.

For eliminating the influence of rate variation in regard to time correction, the writer used the chronometer rate which is obtained from signal receptions for J. Ball and Th. Niethammer longitude values.

At the end, the writer presents a final comparison between the semi-definitive values and his results both in longitude and latitude determinations and in addition to that he explains the superiority of his method with regard to the accuracy and rapidity against J. Ball and Th. Niethammer methods.

PART I

This method was investigated and discussed in principle and detail first in 1785 by Aboe and then in 1808 by Gauss and later on in 1811 by Delambre with a view to determining the longitude and latitude; i.e., the geophysical coordinates of a locality on earth surface.

In addition to the above-mentioned investigations, Knorre extended the application of this method to the case of more than three stars. But due to the primitiveness of the fine mechanics and the drawbacks in the precision techniques of the 19th century, these methods have not been used for a long period, [1] until Chandler's Almucantar and Beck's Nadir instruments were developed.

By using the above-mentioned devices one could get easily one second of arc accuracy in latitude observations; as a result of this Knorre's method was used by travelers, only and particularly on the geophysical expeditions.

Nowadays geodesy is proceeding with such extreme precision limits that the application of astrometry should meet the requirements for determining accurately the locations of Laplace stations of the triangulation chains. Today in practice, the astronomers are using portable high-precision universal theodolites and/or transit instruments with 6.5 to 7.0 cm. free objective lense widths and 21 to 27.5 cm. alidade circle radii and 1 second of arc hang levels. In spite of this, the obtainable accuracy in latitude determination is + 0.15 second of arc and in longitude determination + 0.02 second of time.

As for the determination of the deflections of the vertical, one can maintain the above precision figures as maximum limits.

In order to eliminate the errors of accidental and systematical nature and errors of unknown character as well as the orientation error due to deflection of the plumb line, one can combine longitude, latitude and azimuth observations as pair-wise observation system with the same object and in one instrument set-up.

The general advantages of the applied method and the sufficiency of obtained data for solving the problems of position determination in astrometry will now be described and discussed in detail :

A. Computation of the three equal altitude problem will be carried out using the formulae derived by Gauss. We will now use the cosine theorem., with the signs having the following significances :

$Decl_1$,	$Decl_2$,	$Decl_3$	Declinations of the three stars observed at the time of measurement;
AR_1,	AR_2,	AR_3	Right ascensions of the three stars observed at the time of measurement;
U_1,	U_2,	U_3	Transit moments of the three stars measured with an adjusted chronometer;
t_1,	t_2,	t_3	Hour angles of the three stars observed at the time of measurement;
		Δu	Chronometer stand which is assumed to be fixed and which is supposed to correspond to the middle of the observation epoch;
		φ	Latitude of the observation point;
		z	The common zenith distance of the three stars observed.

Using the cosine theorem,

$$tg\ \varphi = -A_1\sin B_1.\sin (t_1+t_2)/2 + \Delta u + A_1\cos B_1.\cos (t_1+t_2)/2 + \Delta u$$

$$A_1\sin B_1 = \sin(t_2-t_1)/2.\ cotg(Decl_1-Decl_2)/2$$

$$A_1\cos B_1 = \cos(t_2-t_1)/2.\ \ tg(Decl_1+Decl_2)/2$$

$$C_1 = (t_2-t_1)/2 + B_1$$

$$tg\ \varphi = A_1\cos(C_1 + t_1 + \Delta u) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

In the same manner,

$$tg\ \varphi = -A_2\sin B_2.\sin (t_1+t_3)/2+\Delta u + A_2\cos B_2.\cos (t_1+t_3)/2 + \Delta u$$

$$A_2\sin B_2 = \sin(t_3-t_1)/2.\ cotg(Decl_1-Decl_3)/2$$

$$A_2\cos B_2 = \cos(t_3-t_1)/2.\ \ tg(Decl_1+Decl_3)/2$$

$$C_2 = (t_3-t_1)/2 + B_2$$

$$tg\ \varphi = A_2\cos(C_2+t_1+\Delta u) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Using A_1 and A_2, an auxiliary angle z_o can be determined in the following manner :

$$tg\ z_o = A_1/A_2$$

$$tg(45°-z_o) = (A_2-A_1)/(A_1+A_2)$$

Therefore,

$$tg\ s = tg(45°-z_o).\ cotg(c_2-c_1)/2$$

$$s = (c_1+c_2)/2+t_1- \Delta u$$

$$\Delta u = s-(c_1+c_2)/2-t_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

For the determination of the common star zenith distance the formula :

$$\cos z = \sin \varphi \sin Decl + \cos \varphi \cos Decl \cos t \quad \ldots\ldots\ldots\ldots \quad (4)$$

This method which was used during the geographical expeditions by many travelers, was also employed in 19th century for the determination of latitude and time.

For instance, we can mention the observations carried out during the night of 6/7 Oct., 1949 at the Ankara Çiftlik Station by using a Tavistock Universal theodolite with a horizontal circle of 12.7 cm. in diameter and an astrolabe with Wild T_3 prism. Three stars with an azimuth difference of 120° were used for these measurements. These stars belong to the master star catalog and in computing their coordinates, short period terms were taken into account.

The amount of refraction correction was calculated using the barometer and temperature readings.

In order to determine an approximate longitude, observations were carried out between the rhythmical time signals. Besides, the chronometer accuracy, obtained from these, was employed for the computation of the longitude.

The errors made in the readings and due to the refraction effects were too small to be considered in the final results. The other reason for not considering these errors is due to the fact that the above example is given only to illustrate the application of the Gauss's method.

The stars mentioned above are marked by the numbers 664, 743 and 870 in the Catalog FK 3.

To carry out the observations, the instrument is set up in the meridian and the division of the horizontal circle corresponding to the north, is turned until it coincides with the zero mark. Then, chronometer readings are made as the stars pass the hair marks in the telescope.

Using the formulae 2, 3 and 4 given above and the observation readings, the following results are obtained :

Date : 6/7 Oct., 1949 — **Observer : Halim Ulutaş**
Observations point : Ankara Çiftlik Station — $t_n = +12°$ **Celsius**
Instrument: Tavistock Universal $\varphi = 12.7$ cm. — **Bar. = 698 mm.**
Chronometer : Ulysse Nardin No. 2688 — $g_o = +0.^s158$

Star No. 664	Star No. 870	Star No. 743
$U_1 = 18^h 37^m 40.^s8$	$U_2 = 20^h 47^m 00.^s2$	$U_3 = 21^h 21^m 20.^s1$
$AR_1 = 17\ 37\ 13.5$	$AR_2 = 23\ 01\ 20.7$	$AR_3 = 19\ 45\ 08.2$
$t_1 = +1\ 00\ 27.3$	$t_2 = -2\ 14\ 20.5$	$t_3 = +1\ 36\ 11.9$
$= +15°\ 06'\ 49.''5$		

$$(t_2-t_1)/2 = -1^h\ 37^m\ 23.^s9 = -24°\ 20'\ 58.''5$$
$$(t_3-t_1)/2 = +0\ 17\ 52.3 = +4\ 28\ 04.5$$

$$Decl_1 = 68°\ 47'\ 03.''2 \quad Decl_2 = 27°48'\ 42.''6 \quad Decl_3 = 18°\ 24'\ 36.''4$$

$$(Decl_1-Decl_2)/2 = 20°\ 29'\ 10.''3$$
$$(Decl_1+Decl_2)/2 = 48\ 17\ 52.9$$
$$(Decl_1-Decl_3)/2 = 25\ 11\ 13.4$$
$$(Decl_1+Decl_3)/2 = 43\ 35\ 49.8$$

870—664 **743—664**

sin $(t_2-t_1)/2$	9,615 2164n	sin $(t_3-t_1)/2$	8,891 5422
ctg $(Decl_1-Decl_2)/2$	0,427 5641	ctg $(Decl_1-Decl_3)/2$	0,327 6358
$A_1 \sin B_1$	0,042 7805n	$A_2 \sin B_2$	9,219 1780
$\sin B_1$	9,865 4190n	$\sin B_2$	9,235 2614
B_1	= 312°49'00."2	B_2	= 9°53'52."7
cos $(t_2-t_1)/2$	9,959 5407	cos $(t_3-t_1)/2$	9,998 6782
tg $(Decl_1-Decl_2)/2$	0,050 1080	tg $(Decl_1+Decl_3)/2$	9,978 7241
$A_1 \cos B_1$	0,009 6487	$A_2 \cos B_2$	9,977 4023
A_1	0,177 3615	A_2	9,983 9133
C_1	= 288°28'01."7	C_2	= 14°21'57."2

TABLE - I

	$(C_1+C_2)/2$	=	151°	24'	59."4	
	$(C_2-C_1)/2$	= —	137	03	02.2	
	z_0	=	57	21	30.4	
	s	= —	13	14	42.6	
	$t_1+\triangle u$	= —	164	39	42.0	
For $20^h 15^m$	$\triangle u$	= +	0	13	28.5	= + 54.'0
	φ	=	39°	55'	46."4	
Including refraction	z	=	30°	00'	30."6	

The discussion of results obtained by using the Gauss's method

It will be seen that there are noticeable differences between the computed values of observation point, latitude, chronometer stand and the astrolabe values in the later sections. Compared with the Wild T_3 astrolabe measurements, the given differences are —3."2 in the latitude and + 1.88 in the chronometer stand. For these reasons, it is not possible to calculate accurately the latitude and the chronometer stand by using small instruments and applying the three equal altitude method. The errors will be decreased to an extent, by employing larger instruments, but they are so large that no satisfactory results can be expected. With these errors in mind, the developed version of John Ball's astrolabe method and its application in geodesy will now be explained in detail.

B. Definition of astrolabe and its use. — Astrolabe consists of an optical prism and a mirror system arrangements which is used in conjunction with any universal theodolite with 40-50 mm. wide objective and 40-65 fold magnification. Such an instrument arrangement can be used to determine positions with a precision comparable to that of a universal theodolite having 14-21 cm. horizontal circle diameter.

Astrolabes are usually fixed in front of suitable theodolites. Their prisms have, on the average, 60° angle range.

The arrangement described is shown in Fig. 1. As shown, the image of the astronomical target can always be positioned in the vertical plane. Besides, two different images of the same target moving relative to each other can be observed in the ocular.

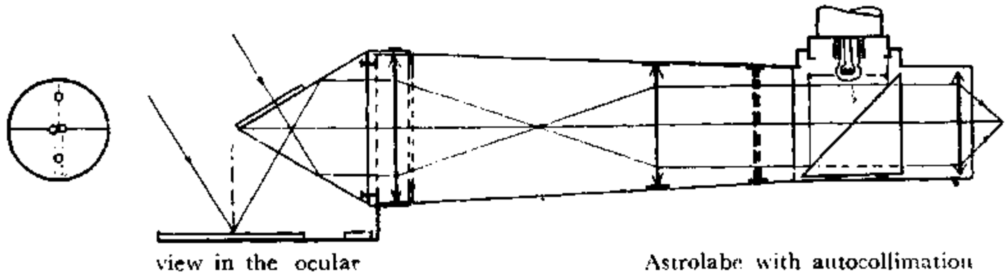


Fig. 1

To determine accurately the position, latitude, and the prism aperture, it would be necessary to fix the moment when the images of two different stars meet. This moment should be determined with care and high precision. It can, for instance, be fixed by the eye-ear, taster, or by means of micrometer methods with $0.^s1$, $0.^s05$, $0.^s01$ accuracies respectively.

In Fig. 2, the most suitable declination limits are indicated for astrolabe measurements between the latitudes of +36° and +43°, with a prism width of 60°.

If we assume that the instrument is set in the meridian, then the DD' horizon corresponding to 60° height angle and the star declinations cutting this line will be greater than j-30° and smaller than j + 30°.

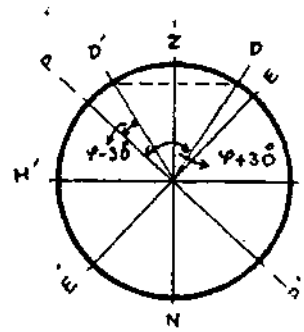


Fig. 2

The limits of average star declinations for our country are assumed to be :

$$10° > \text{declination} > 70°.$$

The observations should be made on both sides of the meridian, using northern and southern stars. Thus, the right ascension limits of the star can be calculated replacing the known quantities in the cosinus theorem.

Since the points D and D' indicate the declination limits, the hour angles corresponding to these points will be minimum zero and maximum 3^h when j = declination.

Therefore,

$$\cos t = 0.8660 \sec. \varphi \sec. \text{Decl.} - \text{tg}\ \varphi\ \text{Decl.} \quad (5)$$

Using this formula, it will be possible to write down the following formulae for the stars west and east of the meridian respectively :

$$\Theta = AR - t$$
$$\Theta = AR + t$$

Hence the star right ascension limits are :

$$\Theta - 3^h > AR > \Theta + 3^h$$

different latitudes. To this end, the latitude of the observation point and the time angles can be determined by varying the declination values in the formula (5) and then using the formula :

$$\sin A = \sec. 60° \cos Decl. \sin t \quad \ldots\ldots (6)$$

The azimuths of the stars can be calculated. This procedure was carried out for Turkey and the following values of star hour angles and azimuths were calculated :

Table of azimuths for z = 30°

φ / φ-Decl.	36°	38°	40°	42°	44°
+ 28°	23°.9	24°.1	24°.4	24°.7	25°.0
+ 26	34.0	34.4	34.7	35.1	35.5
+ 24	41.9	42.3	42.8	43.3	43.8
+ 22	48.7	49.2	49.7	50.3	50.9
+ 20	54.7	55.3	55.9	56.6	57.3
+ 18	60.3	60.9	61.6	62.3	63.0
+ 16	65.6	66.2	66.9	67.6	68.4
+ 14	70.5	71.2	72.0	72.8	73.6
+ 12	75.4	76.1	76.8	77.6	78.5
+ 10	80.0	80.7	81.5	82.3	83.2
+ 8	84.4	85.2	86.0	86.9	87.8
+ 6	88.7	89.5	90.4	91.3	92.2
+ 4	93.0	93.8	94.7	95.6	96.6
+ 2	97.2	98.0	98.8	99.7	100.7
0	101.3	102.1	103.0	103.9	104.9
− 2	105.3	106.2	107.1	108.0	109.0
− 4	109 3	110.2	111.1	112.1	113.1
− 6	113.3	114.2	115.1	116.1	117.1
− 8	117.3	118.2	119.1	120.1	121.1
− 10	121.3	122.2	123.1	124.1	125.1
− 12	125.4	126.3	127.2	128.1	129.1
− 14	129.5	130.3	131.2	132.2	133.2
− 16	133.7	134.5	135.3	136.2	137.2
− 18	137.9	138.7	139.5	140.4	141.3
− 20	142.3	143.0	143.8	144.6	145.5
− 22	146.9	147.6	148.3	149.1	149.9
− 24	152.0	152.6	153.2	153.9	154.7
− 26	157.8	158.4	159.0	159.6	160.3
− 28	164.4	164.8	165.2	165.7	166.2

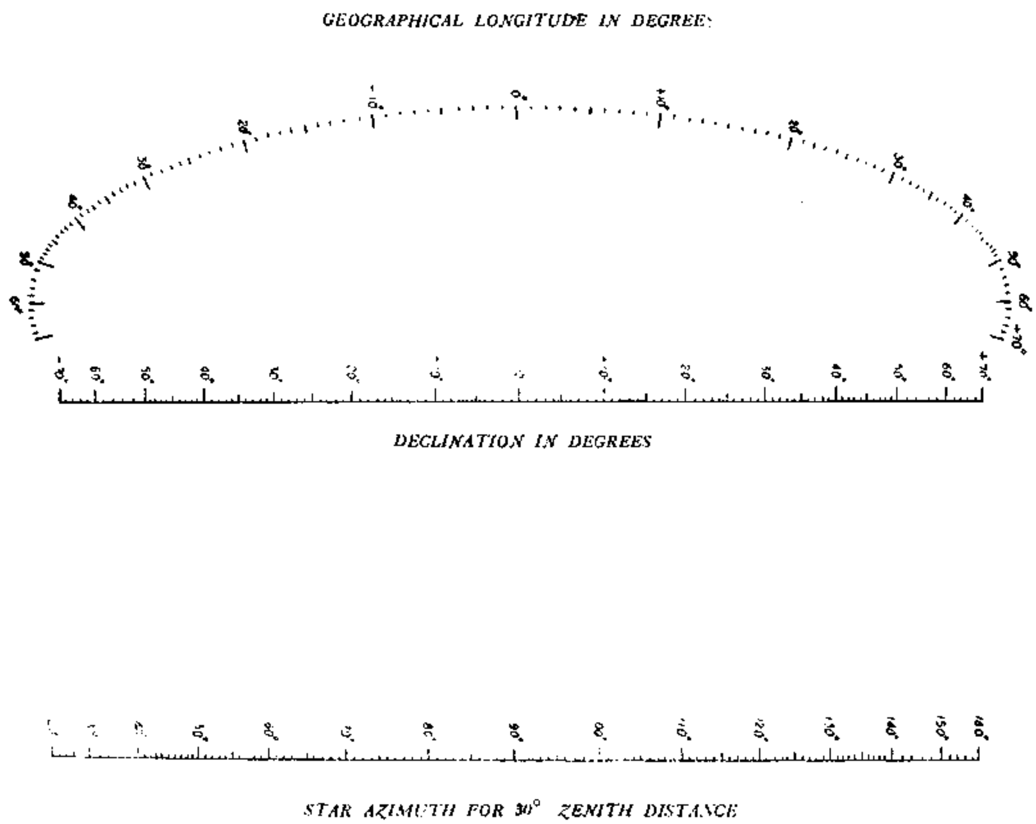
Table of time angles for z = 30°

φ / φ-Decl.	36°	38°	40°	42°	44°
+ 28°	$0^h 47^m$	$0^h 48^m$	$0^h 49^m$	$0^h 50^m$	$0^h 51^m$
+ 26	1 06	1 07	1 08	1 09	1 11
+ 24	1 20	1 21	1 23	1 24	1 26
+ 22	1 31	1 33	1 35	1 37	1 39
+ 20	1 41	1 43	1 45	1 47	1 50
+ 18	1 49	1 51	1 53	1 56	1 59
+ 16	1 56	1 58	2 01	2 04	2 07
+ 14	2 02	2 05	2 08	2 11	2 14
+ 12	2 08	2 11	2 14	2 17	2 21
+ 10	2 12	2 15	2 19	2 23	2 28
+ 8	2 17	2 20	2 24	2 28	2 33
+ 6	2 21	2 25	2 29	2 33	2 38
+ 4	2 24	2 28	2 32	2 37	2 42
+ 2	2 27	2 31	2 35	2 40	2 45
0	2 29	2 33	2 38	2 43	2 49
− 2	2 31	2 35	2 40	2 45	2 51
− 4	2 32	2 36	2 41	2 47	2 53
− 6	2 32	2 37	2 42	2 48	2 55
− 8	2 32	2 37	2 43	2 49	2 56
− 10	2 32	2 37	2 43	2 49	2 56
− 12	2 30	2 36	2 42	2 48	2 55
− 14	2 28	2 33	2 40	2 46	2 54
− 16	2 24	2 30	2 36	2 43	2 51
− 18	2 19	2 25	2 31	2 38	2 46
− 20	2 13	2 19	2 25	2 32	2 40
− 22	2 04	2 10	2 16	2 23	2 31
− 24	1 52	1 57	1 64	1 71	1 79
− 26	1 35	1 40	1 46	1 53	1 60
− 28	1 11	1 15	1 20	1 25	1 31

Using a Wild T3 Universal Theodolite having a magnifying power of 40, it was possible to choose stars with a magnitude of 5.

Adjustment of the instrument and its prism

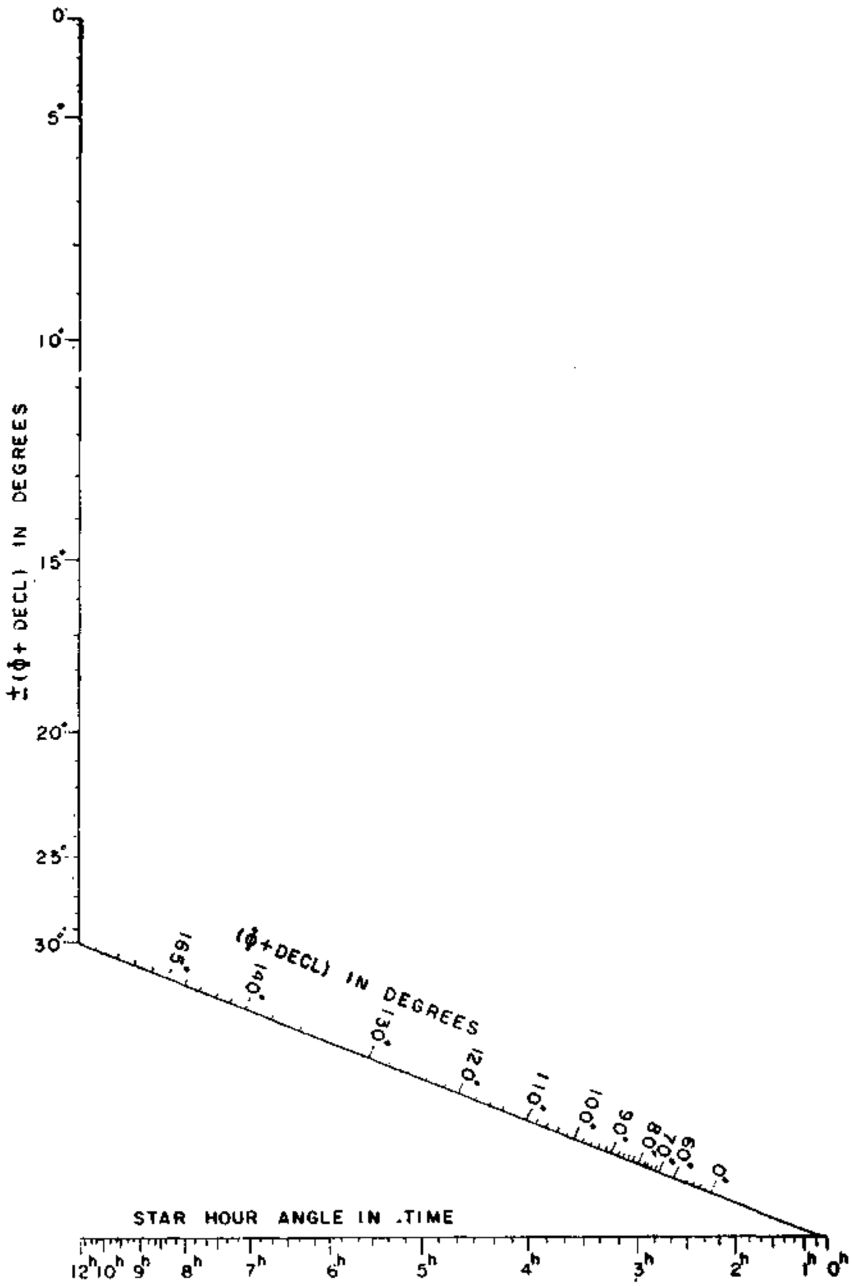
After leveling the instrument, its telescope will be turned to a pole star and/or polaris of which the zenith-distance and azimuth have already been computed. (About computational accuracy of these towarding elements one may



Star azimuth nomogram for 30° zenith distance

refer to the preceding explanations.) Then the telescope with its one side turned about a joint will be moved carefully into a horizontal position. Its general vertical motion screw will then be clamped. After the circular level of astrolabe is corrected with the aid of the vertical motion refining screw, care should be exercised to the adjustment of the parallax of cross-hairs of the telescope by keeping the tube in a 30-degree position with utmost attention. As a next step the upper and lower side faces of the astrolabe will be cleaned and the adjusting mirror attached by a small slide force.[2]

After this procedure, the autocollimation tube will be mounted by replacing the ocular of the main tube,[3] and the autocollimation of the cross-hair will be fixed up, so that the faint image of the main tube and autocollimation tube cross-hairs should both be in good appearance. It could happen sometimes that



Star hour angle nomogram for 30° zenith distance

these images are out of sight and in such a case, slight pressures should be exerted to the edge of the prism in order to have them in sight.

To eliminate the faintness and parallax of the mentioned cioss-hair images, the three adjusting screws of the astrolabe can be turned so that a full coincidence of both cross-hair systems occur simultaneously.

At the end of this procedure, the telescope and its astrolabe will be put together to the horizontal position again and the general vertical clamp screw will be fixed and the adjusting mirror will be removed. For controlling the horizontability of the instrument one can observe and read 0 or 90 degree marks and satisfy the requirements.

After that the autocollimation attachment will be taken out of the instrument and the large magnifying ocular will be screwed again into its proper place. Later the quicksilver will be poured upon the quicksilver pot and stirred by a small glass stick so that the whole quicksilver face may become clean and the pot will be covered by a wind and dust plug.

In this way the instrument is ready for preliminary observation purposes.

A part of the astrolabe observation program for Ankara Çiftlik Longitude Comparison Station

φ — 39° (north)

A	Θ	Cat.	mag.	No.	AR
309° 00'	17h 00m	FK3	4.1	19467	14h 23m
319 30	2	Bos	5.4	21699	16 06
65 15	3	FK3	3.0	27347	19 43
281 45	5	FK3	3.0	19607	14 30
81 00	9	FK3	5.0	27328	19 42
225 00	9	FK3	3.7	21194	15 43
324 30	9	FK3	5.7	20012	14 50
252 15	11	FK3	5.0	20340	15 05
4 15	12	FK3	4.9	Drac.	17 38
101 30	14	Bos	3.2	26953	19 28
257 00	15	Bos	4.7	20285	15 02
294 15	17	FK3	5.4	19747	14 37
191 00	19	FK3	3.4	Ophi.	16 55
109 00	21	Bos	4.6	26904	19 26
226 00	21	Bos	3.9	21408	15 54
234 00	26	FK3	4.3	21255	15 46
62 15	32	FK3	4.0	28099	20 12
137 30	35	FK3	4.2	Aqul.	18 57
167 30	38	Bos	3.7	Ophi.	18 04
271 15	40	FK3	3.5	20523	15 13
264 30	42	Bos	5.1	20696	15 21
75 15	43	FK3	2.3	28338	20 20
256 15	44	FK3	2.3	20917	15 32
262 00	44	FK3	3.7	20795	15 25
323 45	44	Bos	3.5	20747	15 24
142 15	49	FK3	3.0	Aqul.	19 02
279 45	56	FK3	4.5	20724	15 22
65 15	59	FK3	1.3	Cygni	20 39

Bos.: Indicates the stars of Boss Fundamental Star Catalog.

φ — 38° (north)

A	Θ	Cat.	mag.	No.	AR
53° 30'	17h 00m	FK3	4.6	27141	19h 35m
326 30	1	FK3	5.7	20012	14 50
283 15	4	FK3	3.0	19607	14 30
64 00	6	FK3	3.0	27347	19 43
79 30	10	FK3	5.0	27328	19 42
99 45	13	Bos	3.2	26953	19 28
254 00	13	FK3	5.0	20340	15 05
228 15	14	FK3	3.7	21194	15 43
295 00	14	FK3	5.4	19747	14 37
259 00	16	Bos	4.7	20285	15 02
107 15	19	Bos	4.6	26904	19 26
159 00	21	Bos	3.7	Ophi.	18 04
229 00	26	FK3	3.9	21408	15 54
134 15	30	FK3	4.2	Aqul.	18 57
236 30	30	FK3	4.3	21255	15 46
60 30	34	FK3	4.0	28099	20 12
200 15	36	Bos	3.4	Ophi.	16 55
287 30	36	Bos	3.6	20226	15 00
334 30	37	FK3	5.1	21246	15 46
138 30	42	Bos	3.0	Aqul.	19 02
223 15	44	FK3	4.5	Hercl.	16 22
263 45	44	FK3	3.7	20795	15 25
73 45	45	FK3	2.3	28338	20 20
258 30	45	Bos	2.3	20947	15 32
40 30	49	FK3	4.3	Cygni	20 12
126 30	55	FK3	4.5	27236	19 38

FK 3: Indicates the stars of Berliner Astronomisches Jahrbuch.

Execution of the observations

By the aid of time signal controlled chronometer clock and with the observation program star co-ordinates, i.e. siderial time, azimuth and zenith-distance of sky objects, one may promptly direct the instrument upon the star and wait for its exact coincidence time through the middle cross-hair.

With this setup of the instrument one may observe the image of the twinkling sky objects every time as the two stars move towards each other in opposite direction within the field of sight. The coincidence of these two pictures on the middle cross-hair pair corresponds to the desired time and should be measured with 0.1 second of time accuracy.[4] This time will be noted as exactly as it is by the observer (see Fig. 4).

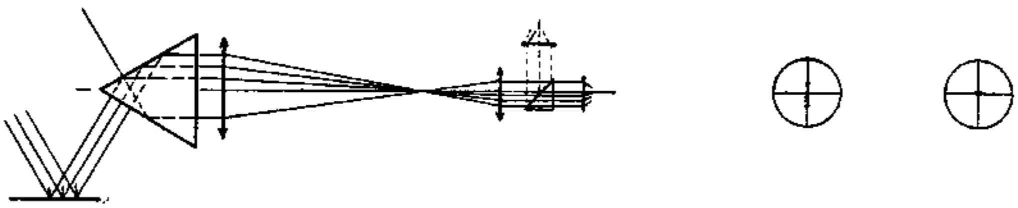


Fig. 4

In this way the coincidence should occur at the middle of the telescope field. If this does not occur, that is to say if the coming together takes place a little above or under of the mentioned intersection line, the stand of the telescope should then be corrected and its position changed by the aid of the fine movement screw of the vertical circle. If the coincidence should occur at the left and/or at the right side of the vertical wire, similarly one can make use of the horizontal fine movement of the azimuth circle of the setup.

If the observer will detect that both images are not running in the same vertical plane during their course, one can make use of the fine movement at the top standing screw of the astrolabe prism. Thus, the overall check-up is completed and the observer can begin his actual survey.

In order to determine the stand of the clock within + 0.1 second of time accuracy, the observer has to take three or four readings for each star group. Observations should carry on successively. In order to find out the stars to be observed in an easy way, one can mark the north point of the azimuth circle with half minute of arc accuracy. Each time attention should be paid to check the setup.

Every observer knows naturally that the azimuth of stars are counting in clockwise direction from the astronomic north.

Note : By using 60-degree equal-side prism one can observe only 60-degree altitute stars, therefore the observer is obliged to set up the azimuth of the object and wait on the passage of it.

The azimuth differences of the consecutive stars to be observed may differ by 90 degrees. For the same reason, differences up to 120 degrees in star azimuths can also be expected. Beyond these limits observation results are not satisfactory.

Dr. John Ball's graphical computation method

By the aid of this method the latitude and the clock stand will be evaluated in a very practical way. Thus the observed stars will first be separated to their groups and in every group a NE, SE, SW and NW star scries should be encountered. Later on by the aid of a geometrical configuration system u, j and Az corrections will be computed.

If, j_o : Indicates the approximate latitude of the observation place,
Z : Indicates the stars zenith-distance (i.e. 30°+ the amount of the vertical refraction),
AR : Indicates the right assension of the star,
Decl : Indicates the declination of the star,

then one may compute the hour angle of the star for the moment of coincidence by the aid of the spherical cosine rule,

$$\cos Z = \sin j \sin Decl + \cos rp \cos Decl \cos t$$

Replacing cost by the following relation :

$$\cos t = 1 - 2 \sin^2 t/2$$

$$\cos Z = \sin \varphi \sin Decl + \cos \varphi \cos Decl (1-2 \sin^2 t/2)$$

$$\cos Z = \sin \varphi \sin Decl + \cos \varphi \cos Decl - 2 \cos \varphi \cos Decl \sin \sin^2 t/2$$

Here, $\cos (\varphi - Decl) = \sin \varphi \sin Decl + \cos \varphi \cos Decl$

$$\cos Z - \cos (\varphi - Decl) = - 2 \cos \varphi \cos Decl \sin^2 t/2$$

then, $\cos Z - \cos (\varphi - Decl) = - 2 \sin (Z+\varphi-Decl)/2. \sin (Z-\varphi+Decl)/2$

$$\sin^2 t/2 = \sec \varphi \sec Decl \sin (Z+\varphi-Decl)/2. \sin (Z-\varphi+Decl)/2$$

since, $2 S = Z+\varphi+Decl$

then, $\sin^2 t/2 = \sec \varphi \sec Decl \sin (S-\varphi) \sin (S-Decl)$ .... (7)

As we used before,

$$\Theta = AR \pm t = U_o + \Delta u = U_i$$ (+) star in the east; (−) star in the west

we finally obtain

$$\Delta u = AR \pm t - U_o \qquad (8)$$

Hence, the u's which were computed using the formula (8), can be adjusted graphically, thus solving the problem.

A straight line in the direction of cast-west is drawn on the computation paper. This line would correspond to the approximate value j_o of the latitude of the observation point. If we assume that the approximate values of the latitude and the zenith-distance are known within one or two arcs of second, then one centimeter would correspond to one second of arc, and one second of time to 15 cos j_o centimeters.

If the approximate values are known roughly, the scale should be reduced in order not to enlarge the figure.

Using the above data and procedure, the straight line can be divided into equal divisions of 1/10 second of time.

If the sign of the stands is positive, this straight line should be marked from left to right, and vice versa. Then the stands are marked with their proper positions on the line, in their order of magnitude.

The perpendiculars from every point u to the mentioned straight line will indicate the astronomical north-south direction. Therefore the azimuth of the stars can be plotted simply by the aid of a protractor in relation to these north-south lines. Also perpendiculars from the same points will be drawn. The direction of the star azimuth will then be represented by the position lines of the stars. Intersections of these position lines with each other should form a triangle and/or a quadrilate.

Note : For the reason of comparison of this method with the other observation methods and due to unexpected weather conditions, such as dust, fog and sudden cloud coverings, one may have to select three each group program stars instead of four each group with 90-degree azimuth differences. Thus the accuracy of results of the observations will be affected under different weights.

The stars with 180-degrec azimuth differences are called one astrolabe pair. Therefore, the pair-wise perpendiculars which are going to be erected from the mids of related position lines will represent a small closed figure. The size of this geometric figure is the measure of accuracy of the observed star groups. If we assume that this figure is a point, then the abscissa and the ordinate of this point will supply us respectively the sought u and j corrections of stand of the chronometer and of the latitude. In addition to this, the mean of the length of the perpendiculars will furnish us directly the correction of the zenith-distance (Fig. 5a).

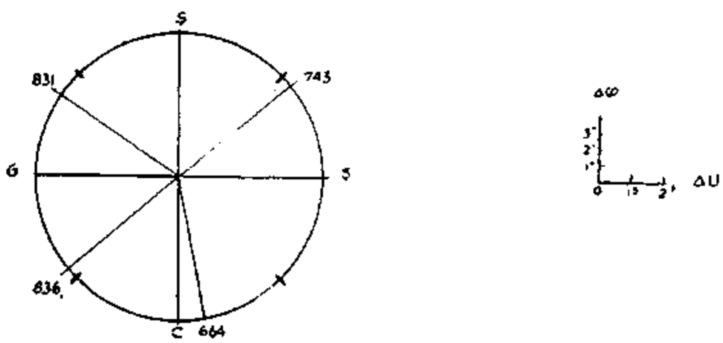


Fig. 5a

The signs of the corrections

The center of the co-ordinate system will correspond to a good approximate value of latitude and chronometer stand correction. According to this we can state that,

a) If the signs of the u values are positive, their places are going to be located always at the right side of the co-ordinate system whether they become large or small.

b) The j values are always positive when they are located above the east-west axis and in contrary they-all carry negative signs.

c) The signs of the z's, astrolabe pairs drawn on position lines, the direction of azimuths are determined by considering their convergence and divergence as to or from the point 0. If the arrows indicate toward 0, the sign of z will be negative, otherwise positive.

Computation of the prism angle

As it will be seen from Fig. 1, this angle, which is the prism head opening in front of the astrolabe base supported by the objective, is 60 degrees. To find its value by the use of Ball group of observations, it would be necessary to add the average z to the zenith-distance corrected for refraction effects.

Example :

Observation Program

Star	Direction	Mag.	AR	Decl.	Azimuth	U_o
664 FK3	SE	4.9	$17^h37^m13.50$	$+68°47'03''.2$	$169°01'$	18^h38^m
836 »	SW	3.6	22 09 07.11	+57 57 20.0	229 32	19 37
831 »	NW	4.0	22 04 40.64	+25 06 02.8	294 20	19 59
743 »	NE	3.9	19 45 08.18	+18 24 36.4	50 45	21 21

Observation point : Ankara Çiftlik Long. Station
Observer : Kasım Yaşar φ_o : $+39°55'43''$
Instrument : No. 13658 Wild T3 Astrolabe Z : 30 00 30.6
Date : 6/7. October, 1949 L_o : — $2^h11^m09^s.7$
Time of observation : 19^h54^m (epoch 13^h06^m) Δu_o : + $54^s.8$
Barometer : 698 mm. (GBR^{12}—GBR^{23}) g_o : + $^s.158$
Thermometer : +12° Celsius

Computation of hour angles and approximate chronometer stands

Sign	664	836	831	743
cos Z sec φ	0,052 7868	0,052 7868	0,052 7868	0,052 7868
sec Decl	0,441 4339	0,275 2512	0,043 0813	0,022 8160
I	0,494 2207	0,328 0380	0,095 8681	0,075 6028
I =	3,120 4748	2,128 2902	1,247 0046	1,190 1503
II = —	2,156 0930	— 1,337 1348	— 0,392 0832	— 0,278 5897
cos t =	0,961 3768	0,791 1554	0,854 9214	0,911 5606
t =	$15°20'20''.8$	$37°42'23''.0$	$31°14'56''.2$	$24°16'41''.0$
tg φ	9,922 7143	9,922 7143	9,922 7143	9,922 7143
tg Decl	0,410 9542	0,203 4609	9,670 6640	9,522 2508

Computation of hour angles and approximate chronometer stands (suite)

Sign		664	836	831	743
II	....	0,333 6685	0,126 1752	9,593 3783	9,444 9651
E -- AR	=	$17^h37^m13^s.50$	$22^h09^m07^s.11$	$22^h04^m40^s.64$	$19^h45^m08^s.18$
W ∓ t	=	+ 1 01 21.39	— 2 30 49.53	— 2 04 59 75	+ 1 37 06.73
AR ∓ t	=	$18^h38^m31^s.89$	$19^h38^m17^s.58$	$19^h59^m40^s.89$	$21^h22^m14^s.91$
U	=	18 37 38.79	19 37 21 88	19 58 45.29	21 21 18.95
Δ u	=	+ 55.93	+ 55.70	+ 55.73	+ 55.96

Graphical computation of u, j and Az (Fig. 56)

If we assume that there exist,

An interval for u, 0.5 cm., 15 cos j_o

» » » j 0.5 »

» » » z 0.5 » sec j_o

then it would be possible to calculate these quantities using the graphical procedure shown below :

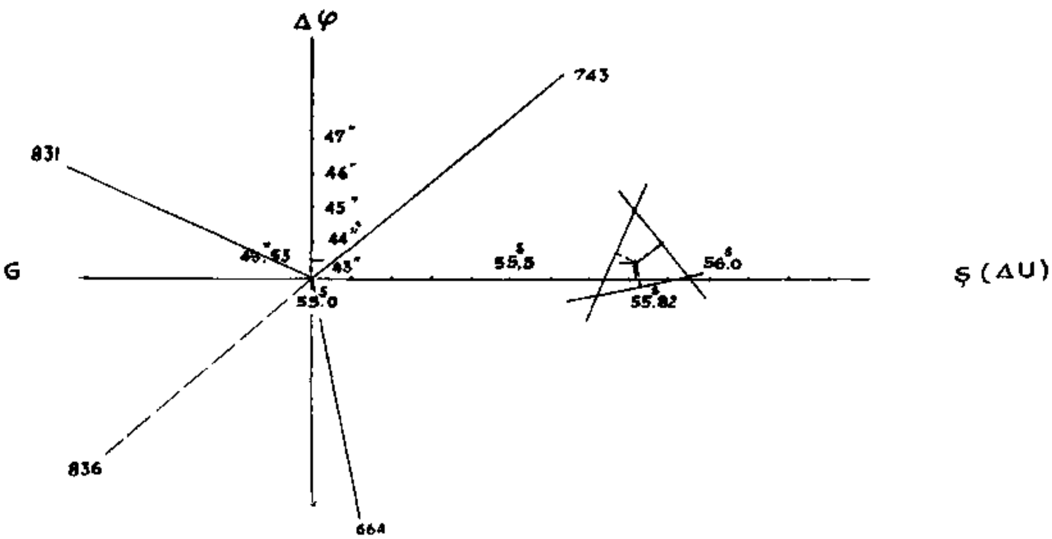


Fig. 56

u = $+55^s.82$

j = +0."53

z = +1/4.3,47.1,3

Thus, the definite values are :

u = $+ 55^s.82$

j = $+39^o55'43.''53$

z = 30°00'31."13

Discussion of the results obtained by the graphical method

The values of latitude, time and the position determined through the calculations are compared with those obtained by Dr. Th. Niethammer's adjustments method which will be explained later. The differences between the two different values are. as follows :

Chr. stand and longitude : (Niethammer - Ball) =—0.^{s}01
Latitude of observation point : (Niethammer - Ball) = —1."29
Zenith distance : (Niethammer - Ball) = —0."90

In the determination of the last difference, variations in the refraction are not taken into account.

It is seen that although the discrepancies in time and longitude stay within narrow limits, those in the latitude and zenith distances are rather large. These large differences are due to the sudden variations in the barometric values and temperature during the observation of the star 639 and to the rather high (—1) value of the star 831.

The azimuth differences in the two programs could not be kept constant at 90° because of cloud hindrance, too strong winds and dust. Thus, it was beyond the writer's means to be able to use two groups of four in both methods and to carry out a satisfactory comparison.

The main and essential purpose of the writer is to compare the known methods among each other and then to explain in detail the results obtained by using his method of observation and computation.

Dr. Niethammer's adjustments method

The following theoretical data and information should be provided with a view to establishing the accuracy in determining quantities by using the approximate values relating to the positions of the instrument and the observation point.

If z is the zenith distance corresponding to the moment when the celestial body crosses the horizontal hair and dr is the amount of correction applied to the average refraction for the zenith distance at the moment of the measurement, then the true value of zenith distance is :

$$z' = z + Ref_o + dr$$

Here let us assume that

$$z'' = z + Ref_o \text{ , then}$$
$$z' = z'' + dr$$

Besides, we can put down the following relations between the approximate and the true values :

$$z'' = z_o'' + dz$$
$$j = j_o + d \quad j \qquad (9)$$
$$u = u_o + d \quad u$$

If we replace the formulae (9) in their respective places in the formulae (4), we can obtain the following relations :

$\cos z''_o - \sin \varphi_o \sin Decl_o - \cos \varphi_o \cos Decl_o \cos t_o + \cos \varphi_o \sin z''_o \sin A_o dt - \sin z''_o \cos A_o d\varphi - \sin z''_o - (dz + dr) = 0$ .... (10)

It is possible to replace the second and third terms of the above equation by $\cos Z_o$. As $dt = d\Delta u$,

then,

$\cos Z''_o - \cos Z_o + \cos \varphi_o \sin Z''_o \sin A_o d\Delta u - \sin Z''_o \cos A_o d\varphi - \sin Z''_o (dz + dr) = 0$

on the other hand,

$$\cos Z''_o - \cos Z_o = -2 \sin (Z''_o + Z_o)/2 . \sin (Z''_o - Z_o)/2$$

As the difference between Z''_o and Z_o is very small the following expression can be written down,

$$\cos Z''_o - \cos Z_o = (Z_o - Z''_o) \sin Z''_o$$

Replacing this in (10),

$(Z_o - Z''_o) \sin Z''_o + \cos \varphi_o \sin A_o \sin Z''_o d\Delta u - \cos A_o \sin Z''_o d\varphi - \sin Z''_o (dz + dr) = 0$

Dividing both sides by $\sin Z''_o$, and carrying out the necessary steps, the error equation can be obtained as follows :

$$\cos \varphi_o \sin A_o d\Delta u - \cos A_o d\varphi + Z_o - Z''_o - dz - dr = 0$$

$$Z_o - Z''_o - dr = 1$$

$$dz + \cos A_o d\varphi - \cos \varphi_o \sin A_o d\Delta u - 1 = v \quad \text{.....................} \quad (11)$$

Note : The v's in the equation (11) are errors and the sum of their squares is a minimum.

The aberration corrections for the star right ascension should be determined in view of the chronometer stand. Thus,

$$d\Delta u = d\Delta u_o - 0^s.021 \cos Z''_o$$

and the error equation for the precise calculations is

$$dz + \cos A_o d\varphi - \cos \varphi_o \sin A_o d\Delta u_o - 1 = v \quad \text{....................} \quad (12)$$

We will now evaluate in detail the astrolabe observations carried out during the night of 6/7 Oct. 1949, with a view to illustrating the practical solution of the problem.

Observation point	: Ankara Çiftlik Longitude Station		$\varphi_o = +39°55'43''$
Instrument	: No. 13658 Wild T3 Astrolabe		$Z''_o = 30°00'30''$
Observer	: Kasım Yaşar		$L_o = -2^h11^m09^s.7$
Date	: 6/7 October, 1949		
Time of observation	: $19^h58^m.3$	(epoch 13^h06^m)	$\Delta u_o = + 54^s.8$
Barometer	: 696.5 mm. (GBR 12 — GBR 20)		$g_o = + 0^s.158$
Thermometer	: +13°.0 Celsius		

Example :

Observation program

Star	Dir.	Mag.	AR	Decl.	Azimuth	U	t
664 FK3	SE	4.9	$17^h 37^m 13.50$	+68° 47′ 03.2	169° 01′	$18^h 37^m 38^s.79$	$+1^h 01^m 21^s.39$
639 »	SE	3.2	17 08 37.02	+65 46 45.1	158 13	18 55 11.86	+1 47 30.56
831 »	NW	4.0	22 04 40.64	+25 06 02.8	294 20	19 58 45.29	—2 04 59.75
676 »	SE	2.4	17 55 25.56	+51 29 48.6	126 16	20 36 04.77	+2 41 35.19
870 »	NW	2.6	23 01 20.70	+27 48 42.6	283 39	20 46 57.73	—2 13 26.96
743 »	NE	3.9	19 45 08.18	+18 24 36.4	50 45	21 21 18.95	+1 37 06.73

Stand comparisons and the approximate calculation of Δu_o

$GBR_{12}h$	U_{1812}	$GBR_{20}h$	U_{1812}	$FYA^2_{22}h$	U_{1812}
1	$13^h 04^m 10^s.3$	1	$21^h 05^m 27^s.3$	1	$23^h 11^m 48^s.2$
62	5 10.4	62	6 27.4	62	12 48.3
123	6 10.5	123	7 27.5	123	13 48.4
184	7 10.7	184	8 27.7	184	14 48.5
245	8 10.8	245	9 27.9	245	15 48.7
306	9 10.8	306	10 28.0	306	16 49.0
153.5	13 06 40.6	153.5	21 07 57.6	153.5	23 14 18.52
Δu^1_o	+ 54.8	Δu^2_o	+ 56.4	Δu^3_o	+ 56.6

Calculation of limits in the error equations

Star		664	639	831	676	870	743
sin φ_o		9.8074218	9.8074218	9.8074218	9.8074218	9.8074218	9.8074218
sin Dekl	...	9.9695205	9.9599811	9.6275827	9.8935253	9.6689152	9.4994348
Pay	...	9.7769423	9.7674029	9.4350045	9.7009471	9.4763370	9.3068566
	=	0.5983321	0.5853328	0.2722726	0.5022813	0.2994588	0.2027013
cos Z_o	=	0.8659534	0.8659498	0.8659627	0.8659516	0.8659498	0.8659508
cos Z_o	...	9.9374945	9.9374927	9.9374992	9.9374936	9.9374927	9.9374932
Z_o	=	29″.75	31″.21	25″.86	30″.49	31″.25	30″.81
cos φ_o	...	9.8847074	9.8847074	9.8847074	9.8847074	9.8847074	9.8847074
cos Decl	...	9.5585661	9.6130530	9.9569187	9.7941797	9.9466903	9.9771839
cos t_o	...	9.9842468	9.9503535	9.9319337	9.8818208	9.9217953	9.9597856
Denominator	...	9.4275203	9.4481139	9.7735598	9.5607079	9.7531930	9.8216769
	=	0.2676214	0.2806170	0.5936901	0.3636703	0.5664910	0.6632495
—1	=	+ 0″.25	— 1″.21	+ 4″.14	— 0″.49	— 1″.25	— 0″.81

Error Equations

$$
\begin{aligned}
dz - 0.98\ d\varphi - 0.19\ \cos\varphi_o\, d\Delta u_o + 0.25 &= v_1 \\
dz - 0.93\ d\varphi - 0.37\ \cos\varphi_o\, d\Delta u_o - 1.21 &= v_2 \\
dz + 0.41\ d\varphi + 0.91\ \cos\varphi_o\, d\Delta u_o + 4.14 &= v_3 \\
dz - 0.59\ d\varphi - 0.81\ \cos\varphi_o\, d\Delta u_o - 0.49 &= v_4 \\
dz + 0.24\ d\varphi + 0.97\ \cos\varphi_o\, d\Delta u_o - 1.25 &= v_5 \\
dz + 0.63\ d\varphi - 0.77\ \cos\varphi_o\, d\Delta u_o - 0.81 &= v_6
\end{aligned}
$$

Normal equations and their solutions

Q_{11}	Q_{12}	Q_{13}				[11]
$6\ dz$	$-\ 1.22\ d\varphi$	$-\ 0.26\ \cos\varphi_o\, d\Delta u_o$	$+\ 2.250$	$-\ 1.000$	$=0$	21.128
dz	$-\ 0.203$	$-\ 0.043$	$+\ 0.375$	$-\ 0.166$		
	$+\ 2.80\ d\varphi$	$+\ 1.13\ \cos\varphi_o\, d\Delta u_o$	$+\ 2.202$	0	$=0$	
	$-\ 0.248$	$-\ 0.053$	$+\ 0.457$	$-\ 0.203$		
		$+\ 3.20\ \cos\varphi_o\, d\Delta u_o$	$+\ 2.815$	0	$=0$	
		$-\ 0.011$	$+\ 0.098$	$-\ 0.043$		
						[11.1]
	$+\ 2.562\ d\varphi$	$+\ 1.077\ \cos\varphi_o\, d\Delta u_o$	$+\ 2.657$	$-\ 0.203$	$=0$	20.283
	$d\varphi$	$+\ 0.420$	$+\ 1.035$	$-\ 0.079$		
		$+\ 3.189\ \cos\varphi_o\, d\Delta u_o$	$+\ 2.913$	$-\ 0.093$	$=0$	
		$-\ 0.451$	$-\ 1.115$	$+\ 0.085$		
						[11.2]
		$+\ 2.738\ \cos\varphi_o\, d\Delta u_o$	$+\ 1.798$	$-\ 0.008$	$=0$	17.531
		$\cos\varphi_o\, d\Delta u_o$	$+\ 0.656$	$-\ 0.003$		
						[11.3]
		$d\Delta u_o = -0.''856,\ Q_{11} = +0.183$				16.351

$$dz = +0.''233 \qquad d\varphi = -0.''759 \qquad d\Delta u_o = -0.^s057 \pm 0.^s1$$

$$d\Delta u = -0.^s057 - 0.^s021 \cos Z_o'' = -0.^s057 - 0.^s018 = -0.^s075 \pm 0.^s1$$

Error calculations

The constants (a) and (b) derived from the time measurements by the use of eye and ear method and the meridian transit of the stars, in 1942 and 1948 as well as the prism head angle errors due to the adjustment unit and the temperature variation, determined by the use of Dr. Niethammer method are given below :

Hearing error constant of the author	:	$a = 0.^s093$
Sighting » » » » »	:	$b = 4.^s7$
Adjustment unit error of the author	:	$m_o = \pm 0.^s065$
Star coordinate error	:	$m_+ = \pm 0.^s02$
Prism angle variation error	:	$m_r = \pm 0.^s02$ (taken from Niethammer)
Magnifying power of theodolite ocular	:	$V =$ 40-fold (for Wild T3)

Thus the error before the adjustment is :

$$m_1^2 = (b/2V)^2 + \cos^2 Decl\ \sin^2 q(a)^2 + m_*^2 + m_r$$

Here, if the coefficient of the second term is taken as the average for the six stars, that is to say 0.093, then :

$$m_1^2 = 0.0035 + 0.0086 + 0.0004 + 0.0004 = 0.0129$$

$$m_1 = \pm 0\overset{s}{.}11$$

the latitude error obtained at the end of the calculations is :

$$m_\varphi^2 = \frac{2}{3} \ .\ 225\ .\ 0.0129 = 1.93$$

$$m_\varphi = \pm 1.''38$$

$$m_{\Delta_u} m\varphi\ ^{15}\ \cos\varphi_o = 1.''38/(15.0,77) = \pm 0\overset{s}{.}12$$

the error in dz due to the adjusment axes of the normal equations can be calculated by the following formula :

$$m_{dz} = \pm m_o\sqrt{Q_{11}} = \pm 0\overset{s}{.}065(15)\sqrt{0.183}$$

$$m_{dz} = \pm 0.''42$$

Δu, φ and Z″ values and their errors

Using the equation (9), we can obtain the following values :

$$\Delta u = + \quad 55\overset{s}{.}81 \pm 0\overset{s}{.}12 \text{ epoch } 16^h 58\overset{m}{.}3$$

$$\varphi = + 39°55'42\overset{''}{.}24 \pm 1\overset{''}{.}38$$

$$Z'' = \quad 30°00'30\overset{''}{.}23 \pm 0\overset{''}{.}42$$

Discussion of results of the least square method

The results of the graphical method have so far been discussed in detail and comparisons of different outstanding points of Dr. Ball's method have been made critically. Meanwhile some other comparisons were also considered relating to the results of Dr. Niethammer's method. We can state here that the previous explanations have bearings on the discussion of this method. Additional considerations about the accuracy of the method pertaining to the azimuth differences of stars and their separations could not be carried out.

It should also be mentioned here that the writer's unit weight mean error, which he computed from a large number of the meridian transit measurements of different declination stars. For preliminary computations, his assumed astrolabe prism top angle variation and its mean error were notable read accurately. The abnormal changes of mean errors in chronometer stand, in latitude correction and in zenith-distance were under functional influence of above-mentioned facts. Accordingly chronometer stand and latitude correction mean errors became high and combined zenith-distance variational mean error became fairly low.

The mean errors in right ascension and in declination of the program stars were extracted from the mean of the mean error of 1949 FK3 Star Catalog. Therefore their magnitudes were small and their influence to the obtained results of accuracy were very little and inconsiderable.

The mean error of the variation of the constant prism angle has been computed according to the formula No. 78a of Dr. Niethammer. See «Die Genauen Methoden Der Astronomisch-Geographischen Ortsbestimmung, 1947» page No. 157. The evaluated value of this error is + 0."27.

It is obvious from page No. 21 that this mean error value is 1.6 times smaller than the value which is computed for dz after the adjustment.

From the results of observations evaluated author's value shows a slight difference of 0.008 time second when it is compared with Dr. Niethammer's accepted value. One may think that this discrepancy ties itself with the observation ability of the author and is the cause of unstability of the instrument used.

PART II

The new method of computation applied by the author and the micrometer used in the measurements

The formula (4) was applied to 3 stars with a view to solve the problem in a more practical way. Neglecting the star declination error, we can write down the related equations in differential form :

$$dz - \cos\varphi \sin A_1 (d\Delta u + dt_1) + \cos A_1 \, d\varphi = 0$$
$$dz - \cos\varphi \sin A_2 (d\Delta u + dt_2) + \cos A_2 \, d\varphi = 0$$
$$dz - \cos\varphi \sin A_3 (d\Delta u + dt_3) + \cos A_3 \, d\varphi = 0$$

Here, $d\Delta u$ and $d\varphi$ can be evaluated by using the above equations and the relationship

$$\cos\varphi \sin A_1 \, dt_1 = l_1$$
$$\cos\varphi \sin A_2 \, dt_2 = l_2$$
$$\cos\varphi \sin A_3 \, dt_3 = l_3$$

since,

$$\begin{aligned} dz - \cos\varphi \sin A_1 \, d\Delta u + \cos A_1 \, d\varphi - l_1 &= 0 \\ dz - \cos\varphi \sin A_2 \, d\Delta u + \cos A_2 \, d\varphi - l_2 &= 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13) \\ dz - \cos\varphi \sin A_3 \, d\Delta u + \cos A_3 \, d\varphi - l_3 &= 0 \end{aligned}$$

then we can obtain,

$$D_{d\Delta u} = \begin{vmatrix} (l_1 - l_2) & (\cos A_1 - \cos A_2) \\ (l_1 - l_3) & (\cos A_1 - \cos A_3) \end{vmatrix} = \begin{aligned} & l_1 (\cos A_2 - \cos A_3) + \\ & + l_2 (\cos A_3 - \cos A_1) + \\ & + l_3 (\cos A_1 - \cos A_2) \end{aligned}$$

$$D_{d\varphi} = \begin{vmatrix} \cos\varphi (\sin A_2 - \sin A_1) & (l_1 - l_2) \\ \cos\varphi (\sin A_3 - \sin A_1) & (l_1 - l_3) \end{vmatrix} = \begin{aligned} \cos\varphi \, [& l_1 (\sin A_2 - \sin A_3) + \\ & + l_2 (\sin A_3 - \sin A_1) + \\ & + l_3 (\sin A_1 - \sin A_2)] \end{aligned}$$

$$D_o = \begin{vmatrix} \cos\varphi (\sin A_2 - \sin A_1) & (\cos A_1 - \cos A_2) \\ \cos\varphi (\sin A_3 - \sin A_1) & (\cos A_1 - \cos A_3) \end{vmatrix} = \begin{aligned} \cos\varphi \, [& \cos A_1 (\sin A_2 - \sin A_3) + \\ & + \cos A_2 (\sin A_3 - \sin A_1) + \\ & + \cos A_3 (\sin A_1 - \sin A_2)] \end{aligned}$$

therefore :

$$d\Delta u = -\cos\varphi \sin A_1/Do \cdot (\cos A_2 - \cos A_3)\, dt_1 -$$
$$- \cos\varphi \sin A_2/Do \cdot (\cos A_3 - \cos A_1)\, dt_2 -$$
$$- \cos\varphi \sin A_3/Do \cdot (\cos A_1 - \cos A_2)\, dt_3$$

and

$$d\varphi = + \cos\varphi \sin A_1/Do \cdot (\sin A_2 - \sin A_3)\, dt_1 +$$
$$+ \cos\varphi \sin A_2/Do \cdot (\sin A_3 - \sin A_1)\, dt_2 +$$
$$+ \cos\varphi \sin A_3/Do \cdot (\sin A_1 - \sin A_2)\, dt_3$$

Since $dt = dz/\cos\varphi \cos A$, we can replace this relationship in the above equations as follows :

$$d\Delta u = -\frac{1}{Do}\,[\,(\cos A_2 - \cos A_3)\, dz_1 - (\cos A_3 - \cos A_1)\, dz_2 - (\cos A_1 - \cos A_2)\, dz_3\,]$$

$$d\varphi = \frac{\cos\varphi}{Do}\,[\,(\sin A_2 - \sin A_3)\, dz_1 + (\sin A_3 - \sin A_1)\, dz_2 + (\sin A_1 - \sin A_2)\, dz_3\,] \qquad \ldots\ldots\ldots\ldots\ldots (14)$$

If we assume that the azimuth differences of the three stars observed are 120 degrees each, then,

$$A_1 = A_2 + 120^\circ \text{ and } A_3 = A_1 + 120^\circ$$

Replacing the following relationship,

$$\sin A_3 - \sin A_1 = 2 \cos (A_3 + A_1)/2 \cdot \sin (A_3 - A_1)/2$$
$$\cos A_3 - \cos A_1 = -2 \sin (A_3 + A_1)/2 \cdot \sin (A_3 - A_1)/2$$

we can obtain,

$$\sin A_1 - \sin A_3 = \sqrt{3} \cdot \cos A_2$$
$$\cos A_1 - \cos A_3 = \sqrt{3} \cdot \sin A_2 \; (+)$$

In the same manner :

$$\cos A_3 - \cos A_2 = \sqrt{3} \cdot \sin A_1$$
$$\cos A_1 - \cos A_3 = \sqrt{3} \cdot \sin A_2$$
$$\cos A_2 - \cos A_1 = \sqrt{3} \cdot \sin A_3$$

$$\sin A_2 - \sin A_3 = \sqrt{3} \cdot \cos A_1$$
$$\sin A_3 - \sin A_1 = \sqrt{3} \cdot \cos A_2$$
$$\sin A_1 - \sin A_2 = \sqrt{3} \cdot \cos A_3$$

For the nominator, we obtain,

$$\sin (A_1 - A_2) + \sin (A_2 - A_3) + \sin (A_3 - A_1) = 3\sqrt{3}/2$$

Replacing these expressions in the equations (14), we get for a special case the following :

$$d\Delta u^s = -\frac{2}{45}(\sin A_1\, dz_1 + \sin A_2\, dz_2 + \sin A_3\, dz_3)\sec\varphi$$
$$d\varphi'' = +\frac{2}{3}(\cos A_1\, dz_1 + \cos A_2\, dz_2 + \cos A_3\, dz_3) \qquad \ldots\ldots\ldots\ldots (15)$$

Note : The proof of the equations marked by (+) is shown below.

Assuming. $A_1 - A_2 = A_2 - A_3 = A_3 - A_1 = 120°$

we get,

$$A_3 = 120° + A_1$$

Replacing it in the following equation,

$$\cos A_3 - \cos A_1 = -2\sin(A_3 + A_1)/2.\ \sin(A_3 - A_1)/2$$

we obtain,

$$\begin{aligned}\cos A_3 - \cos A_1 &= -2\sin(A_3 + A_1)/2 \cdot \sin 60° \\ &= -\sqrt{3}\sin(A_3 + A_1)/2 \\ &= -\sqrt{3}\sin(2A_1 + 120°)/2 \\ &= -\sqrt{3}\sin(A_1 + 60°)\end{aligned}$$

Since, $A_1 = A_2 + 120°$

we get,

$$\cos A_3 - \cos A_1 = -\sqrt{3}\sin(A_2 + 120° + 60°)$$

and finally

$$\begin{aligned}&= -\sqrt{3}\sin(180° + A_2) \\ &= \sqrt{3}\sin A_2\end{aligned}$$

Thus, it is possible to calculate and determine the accurate values of the longitude of the observation point and zenith-distance through the use of correction formula (15). The chronometer stand and the longitude of the observation point can also be derived from the equation system (9).

Determination of the average error of the weight unity and the measurement

In order to determine the average error of the weight unity used in the two functions for the hour stand and the latitude of the observation point, it would be necessary to make use of a large number of hair or figure coincidences as in the case of astrolabe measurements. According to Th. Albrecht, the weight unity error for a horizontal hair transit is :

$$m_o = \pm\sqrt{a^2 + (b/V)^2\cos^2\delta\,\sin^2 A}$$

If we consider the error due to the variation of refraction coefficient of the prism angle due to changes in temperature, then the new weight unity error becomes,

$$m = \pm\sqrt{m_o^2 + m_r^2}$$

As,

$$B = b/2V \text{ and } c = a/B$$

then

$$m = \pm \sqrt{B^2 (c^2 + \sec^2 \delta / \sin^2 A) + m_{dz}^2}$$

Assuming that

$$c^2 = \left(\frac{2}{45}\right)^2 \text{ and } c'^2 = \left(\frac{2}{3}\right)^2$$

we can find the average errors in $d\Delta u$ and $d\varphi$:

$$m_{d\Delta u}^2 = c^2 (\sin^2 A_1 . m_{dz_1}^2 + \sin^2 A_2 . m_{dz_2}^2 + \sin^2 A_3 . m_{dz_3}^2) \sec^2 \varphi$$
$$m_{d\varphi}^2 = c'^2 (\cos^2 A_1 . m_{dz_1}^2 + \cos^2 A_2 . m_{dz_2}^2 + \cos^2 A_3 . m_{dz_3}^2) \quad \ldots\ldots (16)$$

Knowing that the sum of sinus's of the three 120 degree azimuth angles is equal to 3/2,

$$m_{dz_1} = m_{dz_2} = m_{dz_3} = m$$

$$(15)^2 \cdot m_{d\Delta u}^2 = \frac{2}{3} m^2 \sec^2 \varphi$$

$$m_{d\varphi}^2 = \frac{2}{3} m^2$$

Combining these two equations we obtain,

$$m_{d\varphi} = 15 \cos \varphi \, m_{d\Delta u} = \sqrt{\frac{2}{3}} m$$

Now, assuming that the quantities m_r used by Dr. Niethammer are small, we can replace these by the values, $m_{dz} = \pm 0.''42 = \pm 0^s.028$ obtained by the author's own observations. In this way, the errors of the chronometer stand and the longitude of the observation point can be calculated as follows :

$$m_{d\varphi}^2 = (15)^2 \cos^2 \varphi \, m_{d\Delta u}^2 = \frac{2}{3N} B^2 (c_o^2 + \sec^2 \delta / \sin^2 A) + m_{dz}^2 \quad \ldots\ldots\ldots (17)$$

Replacing the following quantities calculated on the previous assumptions in formula (17) :

$$\sin^2 A_v = \frac{1}{n} \sum_1^n \sin^2 A = 0.553$$

$$B = 0^s.0588$$
$$c_o = 1.58$$

we can obtain :

$$m_{d\varphi}^2 = (15)^2 \cos^2 \varphi \, m_{d\Delta u}^2 = \frac{1}{N} (0^s.00314 + 0^s.00205 \sec^2 \delta)$$

therefore it is possible to write down the following expression :

$$m_{d\varphi} = 15 \cos \varphi \, m_{d\Delta u} = \pm \sqrt{\frac{1}{N} (0^s.00314 + 0^s.00205 \sec^2 \delta)} \quad \ldots\ldots (18)$$

The errors m_{dj} and $m_{d\ u}$ calculated for different values of N and j are shown in the following two tables :

$m_{d\varphi}$

N	1	2	3	4	5
φ	3+	6+	9+	12+	15+
36°	1.″19	0.″84	0.″68	0.″60	0.″54
37	1.20	0.85	0.68	0.60	0.54
38	1.20	0.85	0.68	0.60	0.54
39	1.21	0.86	0.70	0.61	0.55
40	1.21	0.87	0.70	0.62	0.55
41	1.23	0.87	0.71	0.62	0.56
42	1.23	0.88	0.72	0.63	0.56

$m_{d\Delta u}$

N	1	2	3	4	5
φ	3+	6+	9+	12+	15+
36°	0.ˢ10	0.ˢ07	0.ˢ06	0.ˢ05	0.ˢ04
37	0.10	0.07	0.06	0.05	0.04
38	0.10	0.07	0.06	0.05	0.05
39	0.10	0.07	0.06	0.05	0.05
40	0.11	0.08	0.06	0.05	0.05
41	0.11	0.08	0.06	0.06	0.05
42	0.11	0.08	0.07	0.06	0.05

The values of the errors in these tables are relative, due to the fact that their calculation is based on the eye-ear method of observation, on the meridian transit of the stars and on the (a) and (b) quantities deduced through the time measurements. Considering the values in the table with the errors obtained after the adjustment in the previous pages, we can see that there is a difference of 0.04 second of time in the stand and 0.51 second of arc in j. This indicates that the values in the tables are more theoretical.

The amount of errors which are represented formerly with two separate tables will be diminished when, a Wild T3 astrolabe with a larger dimension is used. The telescope of this set should be furnished in this case with 50mm. free aperture objective lense and with 65 times magnifying power ocular and with an ocular micrometer which has symmetrical wires on its cross-hair plane.

The new ocular outfit for increasing the precision of the observations

After finishing his exprimental observations in September 1949 the author considered the accuracy which could be obtained when his formulae are employed in all the observations. To this end he computed his test measurements which were carried out by Wild T3 astrolabe set. Then he discussed the results and decided to make some changes in the ocular system of his instrument. He also planned to observe and evaluate progressive measurements in October 6/7, 1949.

To every astronomer it is known that, at the middle part of the see-field of an astrolabe one can observe only one figure of coincidence and no other is existing, but instead of this coincidence one can arrange several symmetrical intersections and by the help of the mean of these it is obvious to improve this coincidence instant for every star. For this reason it is necessary to place into the focal distance of the telescope tube a cross-hair carrier which must have several horizontal symmetric, hairs.

The simple cross-hair device of which the author has made use during his measurements was the cross-hair system of one available «Max Hildebrandt Theodolite» mounted back to the Wild T3 Theodolite. Later on the whole mechanism has been adjusted to the geometrical axis of the system.

To understand how to operate with this new ocular system, we may think first of all that the cross-hair attachment should carry only one single hair at the upper and lower part of the middle hair.

Regarding Figure 6, the line (f) should indicate the lower hair and (O) represent the center of coincidence. The images of star will be composed in two different ways. One of them is the refracting image through the prism, the other one is the reflecting and refracting image through the quicksilver mirror and the prism. Thus (OA) and (OA') are the trace images of the star relating to the above-mentioned case.

The sides OA and OA' of the triangle OAA' are equal to each other and for this reason the mean of transit times of both images which were built up through prism and mirror-prism combination, should give us the time of the real single coincidence.

The new cross-hair system has six horizontal hairs at the upper and lower part of the see-field as seen from Figure 7. In addition to these there are pair-wise two horizontal and two vertical hair outfits just in the mids and which cross each other in a square shape.

It will be understood from the explanations that the angles between motion traces of the moving objects and the horizontal hairs will represent the azimuth of the stars.

a. The stars in the prime vertical and in its vicinity will intersect all the horizontal hairs in their course.
b. The stars in the meridian and in its vicinity will not cross any hair in their course.
c. The stars between the above-mentioned limits will intersect some of the hairs and some not.

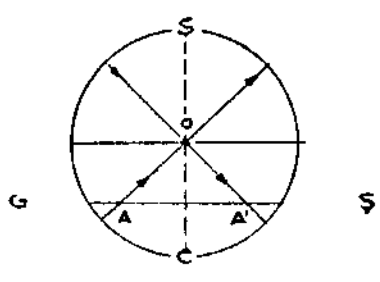


Fig. 6

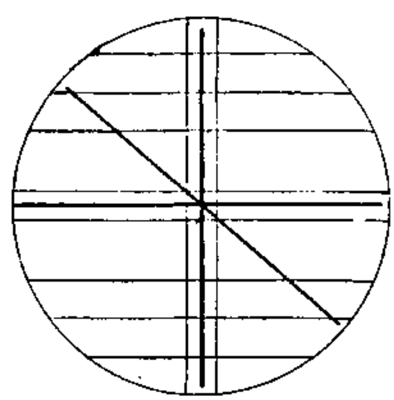


Fig. 7

If the horizontal cross-hairs are not exactly horizontal, the mean of times of the images of the stars through the prism and the mirror will not be the same.

As seen from Figure 8 this mean time could naturally not be the exact coincidence moment.

In order to eliminate this unfavorable situation the author made an attempt at it and arranged another revolving cios-hair carrier ring which has been screwed into an outer ring in such a manner that the whole system could be turned by an eccentric screw attachment from outside the telescope tube.

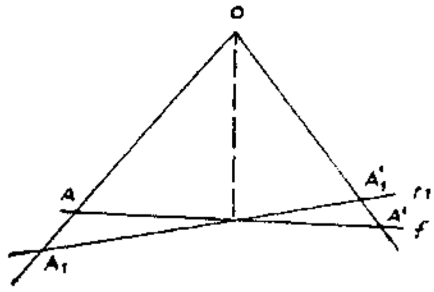


Fig. 8

In this way the observer could continuously be able to correct the horizontabity of the cross-hair by using this screw device and therefore could check the exact hair transits. From the above explanations and from the Figure 9 one can understand without difficulty that the symmetrical observations which were carried out on both sides of the coincidence center will certainly be free from the accidental and systematical errors.

Although this new cross-hair attachment has 6 hairs, the author used in his measurements the four ones nearest to the conincidence center and employed the intersecting times for evaluating the exact coincidences.

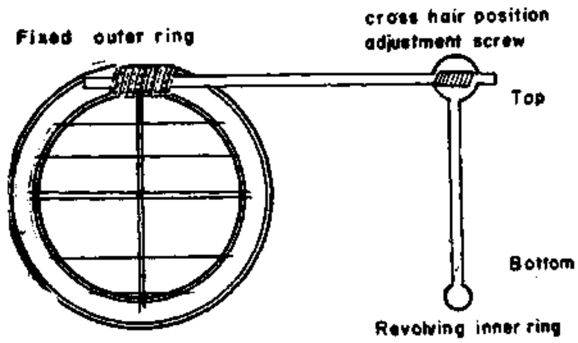


Fig. 9

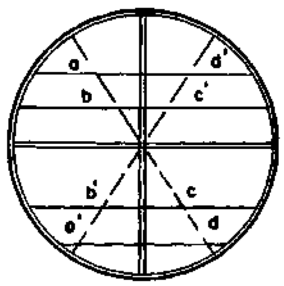


Fig. 10

The above-mentioned 4 hairs are located at the suitable part of the see-field of the telescope and the other two have not been used for observations because of light dispersion and shape deformation of the observed bodies (see Figure 10).

When the upper and the lower hair transit times both are lepresented with the letter (U), the formulae derived by the author are as follows:

The upper coincidence	times	for	hair	No. 1	$1/2\ (U_a + U_{d'}) = U_o^1$
The lower	»	»	»	» No. 2	$1/2\ (U_b + U_{c'}) = U_o^2$
» »	»	»	»	» No. 3	$1/2\ (U_{b'} + U_c) = U_o^3$
» »	»	»	»	» No. 4	$1/2\ (U_{a'} + U_d) = U_o^4$

In order to evaluate the astrolabe measurements carried out during the night of 6/7, Oct. 1962, the necessary observations are given in the following-tables :

Measurements made with the new micrometer

View	Hair No.	U_{664}	U_{639}	U_{831}	U_{870}	U_{743}
Bottom	4 a'	$18^h36^m50^s.6$	$18^h54^m29^s.4$	$19^h58^m26^s.0$	$20^h46^m39^s.0$	$21^h21^m00^s.5$
Bottom	3 b'	37 04.5	54 41.6	58 31.5	46 43.7	21 05.8
Top	1 a	37 04.5	54 41.6	58 31.5	46 43.6	21 05.8
Top	2 b	37 18.3	54 53.8	58 37.1	46 49.3	21 11.1
Top	2 c'	37 59.3	55 29.9	58 53.5	47 06.1	21 26.8
Top	1 d'	38 13.0	55 42.1	58 59.0	47 11.8	21 32.1
Bottom	3 c	38 13.0	55 42.1	58 59.0	47 11.8	21 32.1
Bottom	4 d	38 27.0	55 54.4	59 04.6	47 17.5	21 37.4
Ocular	Coinc.	18 37 38.79	18 55 11.86	19 58 45.29	20 46 57.73	21 21 18.95

Epoch : $19^h55^m.6$

Calculation of prism angle variation

Star		664	639	831	870	743
sin φ	=	0.641 8324	0.641 8324	0.641 8324	0.641 8324	0.641 8324
sin Decl	=	0.932 2244	0.911 9711	0.424 2118	0.466 5683	0.315 8164
Nominator	=	0.598 3321	0.585 3328	0.272 2726	0.299 4588	0.202 7013
cos Z_o	=	0.865 9534	0.865 9498	0.865 9627	0.865 9498	0.865 9508
Z_o	=	30°00'29."75	30°00'31."21	30°00'25."86	30°00'31."25	30°00'30."81
d_z	=	+ 0."25	— 1."21	+ 4."14	— 1."25	— 0."81
cos φ	=	0.766 8447	0.766 8447	0.766 8447	0.766 8447	0.766 8447
cos Decl	=	0.361 8812	0.410 2542	0.905 5631	0.884 4847	0.948 8202
cos t	=	0.964 3769	0.891 9767	0.854 9363	0.835 2092	0.911 5606
Denominator	=	0.267 6214	0.280 6170	0.593 6901	0.566 4910	0.663 2495

If we replace the prism angle variation determined through time measurements corresponding to average coincidence in formula (15), the stand and latitude corrections for the star groups with catalog number 664-831-743 and 639-870-743 would be obtained as follows:

For the Group No. 1

$$d\Delta u^s = -\frac{2.608}{45}\left[(+0.1905)\ (+0.25) + (-0.9112)\ (+4.14) + (+0.7744)\ (-0.81)\right]$$

$$d\varphi = +\frac{2}{3}\left[(-0.9817)\ (+0.25) + (+0.4120)\ (+4.14) + (+0.6327)\ (-0.81\right]$$

For the Group No. 2

$$d\Delta u^s = -\frac{2.608}{45}\left[(+0.3711)\ (-1.21) + (-0.9718)\ (-1.25) + (+0.7744)\ (-0.81)\right]$$

$$d\varphi = +\frac{2}{3}\left[(-0.9286)\ (-1.21) + (+0.2360)\ (-1.25) + (+0.6327)\ (-0.81)\right]$$

the corrections for the two star groups are :

$$d\Delta u_1 = +0^s.25 \qquad d\varphi_1 = +0.''32$$
$$d\Delta u_2 = -0^s.01 \qquad d\varphi_2 = +0.''11$$

considering the aberration correction for the star right ascension, which affects the chronometer standee finally obtain:

$$d\Delta u = +0^s.12 - 0.021 \cos Z_o = +0^s.10 \pm 0^s.08$$
$$d\varphi = +0.''22 \pm 0.''87$$

The discussion of results obtained by the application of the author's new method

The two old methods arc compared with the newly developed method from the point of view of geodetic astronomy, i.e. determination of latitude, stand of the chronometer and of prism top angle values and of their attainable accuracies.

With each of the three methods, computed data and their precisions are put together in the following comparison table.

If these data, concerning each method, will be compared with the semi-definitive values obtained at «Ankara Çiftlik longitude station»; which were carried out by the aid of Askania and Wild made 70 to 65 mm. free lense aperture objective transit instruments by experienced observers, one can immediately see and judge that those three methods are accurate enough for the practical purposes of today's secondary projects of geodesy.

In order to determine dj, u and dz and their mean errors within the limits of the mentioned accuracy, it would also be necessary to take into account the number of groups of stars observed. Thus with Niethammer's and the author's method at least five groups of three each single group stars will be observed in one night and in contrary to that in Ball's graphical method the minimum amount of observation stars may be eight groups of three each single group.

Regarding the explanations given in the earlier pages one can compare the results of two groups of three each single group stars.

Table of Comparison

Author Signs and Names	Th. Niethammer	J. Ball	Kasım Yaşar
Date	6/7 Oct. 1949	6/7 Oct. 1949	6/7 Oct. 1949
Observation epochs	$19^h 58^s.3$	$19^h 54^m$	$19^h 55^s.6$
Amount of observed stars	6	4	6
» » » star groups	2	1	2
Instrument used and its type	Wild T3 Astrolabe with prism	Wild T3 Astrolabe with prism	Wild T3 Astrolabe with prism
Chronometer used	U. Nardin contact chronometer	U. Nardin contact chronometer	U. Nardin contact chronometer
Watch stand and its mean error	$+ 55^s.81 \mp 0^s.12$	$+ 55^s.82$	$+ 55^s.98 \mp 0^s.08$

Table of Comparison (suite)

Authot Signs and Names	Th. Niethammer	J. Ball	Kasım Yaşar
Latitude correction and its mean error	$+ 0''.24 + 1''.38$	$+ 0''.53$	$+ 0''.22 + 0''.87$
Astrolabe prism's top angle correction and its mean error	$+ 0''.23 + 0''.42$	$+ 1''.13$	$+ 0''.22 + 1''.02$
Latitude of observ. station and its mean error	$39°55'13''.21 + 1''.38$	$39°55'13''.53 +$	$39°55'43''.22 + 0''.87$
Prism's top angle and its mean error	$30°00'30''.23 + 0''.42$	$30°00'31''.13 +$	$30°00'30''.22 + 1''.02$

When the first and third column data of the above presented table are thoroughly discussed, it will be seen that, the determined values and their errors calculated through the above-mentioned methods, arc in fairly good agreement. In the first instance, this comparison indicates to the reliability of the results of the author's method. By increasing the quantity of observed star groups one may understand that each described method produces the same basic result. I can therefore say that the developed method can be considered as efficient and accurate as those of J. Ball and Niethammer.

Determination of chronometer rates, weight unity, longitude computation and general comparisons

It is evident that the rate of the chronometer used in the measurements can be determined either from the rhythmical time signals or from the star observations. It is however necessary to apply an adjustment to the g_s values by means of two interval signals. The same applies to g_* obtained through time observations.

If we assume that the average error incurred for a single signal observation is $\pm\ 0^s.06$ and that the weight unity error for the calculation of P_* and P_s corresponding to the rates in question is roughly $\pm\ 0^s.10$, then the rate to be used in the longitude calculations is :

$$g_0\ (P_s\ g_s + P_*\ g_*)/(P_s + P_*)\ \dots\dots \quad (19)$$

The average error of the rate can be determined by,

$$m_{g_0} = 0^s.1/\sqrt{(p_s + p_*)}\ \dots\dots \quad (20)$$

Note: a) Detailed information on the chronometer rates obtained through rhythmical signals can be found in the author's book (in Turkish) entitled «Geodetic Astronomy», published by the Army Map Service.

b) The average signal error was calculated using the rhythmics from different time services of different countries between 1942 and 1949.

If we assume that the stand for the observation epoch is expressed by $\Delta U_{Ep} = \Delta U_v + x = \Delta U_i + dg_i$ and that the chronometer rate correction is indicated by $dg_i = \Delta T_i . g_*$, the correction equation for each result becomes,

$$x - \Delta T_i . g_* - l_i = v_i \quad \ldots \quad (21)$$

where $l_i = \Delta U_v - \Delta U_i$ is the measurement discrepancy.

Thus by adjusting the observations made in the night of 6/7 Oct. 1949, we can obtain the unknown quantities in the formulae (19) and (20).

Error equations

Date	Number of star	Observation epoch	ΔT_i	ΔU_i	$-l_i$	Error equations
6/7 Oct.1949	4	19^h54^m0	-1^m97	$+55^s82$	$+0^s05$	$x+0.0328\,g_* +0.05=v_1$
» » »	6	19 55 . 6	−0 . 37	+55.98	−0,11	$x+0.0061\,g_* -0.11=v_2$
» » »	6	19 58 . 3	+2 . 33	+55.81	+0.06	$x-0.0388\,g_* +0.06=v_3$
		19 55 . 97	−0 . 01	+55.87		

Normal equations

$$\begin{aligned} 3\,x + 0.0001\ g_* + 0.0000 &= 0 \\ x + 0.00003\ g_* + 0.0000 & \\ + 0.0026\ g_* - 0.0014 &= 0 \\ - 0.0000\ g_* - 0.0000 & \\ \hline 0.0026\ g_* - 0.0014 &= 0 \end{aligned}$$

From above we obtain: $g_* = +0^s\!538 \pm 0^s\!1$, $x = -0^s\!00002 \pm 0^s\!07$ and $\Delta U_{Ep} = +55^s\!87 \pm 0^s\!07$

Using the first two values and their errors we can get :

$$m_{g_*} = \frac{m_*}{\sqrt{(bb.1)}} = \frac{0^s\!1}{\sqrt{0.0026}} = \pm 1^s\!96$$

$$p_* = \frac{m_*^2}{m_{g_*}^2} = \frac{0.01}{(1.96)^2} = 0.0026$$

Employing the $GBR_{12}h$ and $FYA^2_{22}h$ signals, two external signals and the time difference of 10.1 hrs., we can compute the following quantities :

$$m_{g_s} = m_s . \sqrt{\frac{i}{(\Delta T)^2}} = 0^s\!06 \sqrt{\frac{2}{(10.1)^2}} = \pm 0^s\!0087$$

$$p_s = \frac{m_s^2}{m_{g_s}^2} = \frac{0.01}{(0.009)^2} = 115$$

$$g_o = \frac{0^s\!158 \times 115 + 0^s\!538 \times 0.0026}{115.0026} = 0^s\!142$$

$$m_{g_o} = \frac{0^s\!1}{\sqrt{115}} = \pm 0^s\!01$$

Time graphs, stand corrections and the determination of approximate longitudes

The time stands determined for a certain moment in each night of observation can be reduced to signal epochs through the following formula :

$$\Delta u_{R.\,Ep} + g.\Delta T = \Delta u_{R.\,Ep} + \Delta u^2$$

In the above formula, the second term represents the stand correction. The computation of these corrections are shown below in three graphs; first two of them were deduced from the external time signals and the third was evaluated through the rates of chronometers determined by an adjustment calculation.

Stand corrections after Th. Niethammer

$g_s = +0\overset{s}{.}158$

$GBR_{12}h$	; S.Ep.	$13^h06^m\!.68$
		$6^h\!.86030$
		$-1^s\!.08 = \Delta u^2$
ΔU	; R.Ep.	$19^h58^m\!.30$
		$1^h\!.16101$
		$+0^s\!.18 = \Delta u^2$
$GBR_{20}h$	; S.Ep.	$21^h07^m\!.97$
		$3^h\!.26683$
		$+0^s\!.52 = \Delta u^2$
$FYA^2_{22}h$	; S.Ep.	$23^h14^m\!.31$

Stand corrections after John Ball

$g_s = +0\overset{s}{.}158$

$GBR_{12}h$	; S.Ep.	$13^h06^m\!.68$
		$6^h\!.78750$
		$-1^s\!.07 = \Delta u^2$
ΔU	; R.Ep.	$19^h54^m\!.00$
		$1^h\!.23280$
		$+0^s\!.19 = \Delta u^2$
$GBR_{20}h$	; S.Ep.	$21^h07^m\!.97$
		$3^h\!.33850$
		$+0^s\!.53 = \Delta u^2$
$FYA^2_{22}h$	; S.Ep.	$23^h14^m\!.31$

Stand corrections after the author

$g_o = +0\overset{s}{.}142$

$GBR_{12}h$	; S.Ep.	$13^h06^m\!.68$
		$6^h\!.81530$
		$-0^s\!.97 = \Delta u^2$
ΔU	; R.Ep.	$19^h55^m\!.60$
		$1^h\!.20620$
		$+0^s\!.17 = \Delta u^2$
$GBR_{20}h$	; S.Ep.	$21^h07^m\!.97$
		$3^h\!.31180$
		$+0^s\!.47 = \Delta u^2$
$FYA^2_{22}h$	; S.Ep.	$23^h14^m\!.31$

Thus, it is only possible to calculate the longitude of the observation point for a single night through the following formula by using the time signals at different epochs and the graphs above :

$$L = (U_{pos} + \Delta U_{R.\,Ep.} + \Delta u^2) - (U_{Grw} + \eta) \qquad \ldots\ldots (22)$$

Here,

$U_{pos.}$ is the sidereal time read on the chronometer at the observation point,

$\Delta U_{R.\,Ep.}$ is the chronometer stand for a certain epoch in the time determination,

ΔU^2 is the stand correction,

U_{Grw} is the amount of time signals as compared with the Greenwich sidereal time,

η is the signal correction due to finite time delay of the electromagnetic waves.

In the following tables, a comparison is made between the approximate longitudes for a single night found by using the time values of Ball and Niethammer and those determined from the author's values :

The approximate longitudes for a single night after John Ball :

Signal	$GBR_{12}h$	$GBR_{20}h$	$FYA^2_{22}h$
$U_{pos.}$	$13^h06^m40^s.60$	$21^h07^m57^s.60$	$23^h14^m18^s.52$
$\Delta U_{R.Ep.}$	+ 55.82	+ 55.82	+ 55.82
Δu^2	— 1.07	+ 0.19	+ 0.53
η	— 0.01	— 0.01	— 0.01
$U'_{pos.}$	13 07 35.34	21 08 53.60	23 15 14.86
$U_{Grw.}$	10 56 25.63	18 57 44.49	21 04 05.18
L	2 11 09.71	2 11 09.11	2 11 09.68
L_v	$2^h11^m09^s.50 \pm 0^s.20 \sec\varphi$		

The approximate longitudes for a single night after Th. Niethammer :

Signal	$GBR_{12}h$	$GBR_{20}h$	$FYA^2_{22}h$
$U_{pos.}$	$13^h06^m40^s.60$	$21^h07^m57^s.60$	$23^h14^m18^s.52$
$\Delta U_{R.Ep.}$	+ 55.81	+ 55.81	+ 55.81
Δu^2	— 1.08	+ 0.18	+ 0.52
η	— 0.01	— 0.01	— 0.01
$U'_{pos.}$	13 07 35.32	21 08 53.58	23 15 14.84
$U_{Grw.}$	10 56 25.63	18 57 44.49	21 04 05.18
L	2 11 09.69	2 11 09.09	2 11 09.66
L_v	$2^h11^m09^s.48 \pm 0^s.20 \sec\varphi$		

The approximate longitudes for a single night after the author :

Signal	$GBR_{12}h$	$Gb_{20}h$	$FYA^2_{22}h$
$U_{pos.}$	$13^h06^m40^s.60$	$21^h07^m57^s.60$	$23^h14^m18^s.52$
$\Delta U_{R.Ep.}$	+ 55.98	+ 55.98	+ 55.98
Δu^3	— 0.97	+ 0.17	+ 0.47
η	— 0.01	— 0.01	— 0.01
$U'_{pos.}$	13 07 35.60	21 08 53.74	23 15 14.96
$U_{Grw.}$	10 56 25.63	18 57 44.49	21 04 05.18
L	2 11 09.97	2 11 09.25	2 11 09.78
L_v	$2^h11^m09^s.67 \pm 0^s.073$ sec φ (*)		

Note: (*) In order to find out about the accuracy with which the longitudes were determined by the author and compare the longitude values with the semi-definite results as shown above, the errors in the rate of chronometer, in the signals and prism angles were used for calculating the error in the longitude.

In fact, the total error in the determined value of the longitude for a single night is equal just to the error in the chronometer stand which consists of the average individual errors in the rate of chronometer in the signal and in the prism angle. This total error can be calculated through the following formula :

$$m_1 = \sqrt{[mg_o\ (E_p.\ S.—E_p.\ U)]^2 + m_s^2 + m_{dz}^2}$$

If we replace the quantities in this formula by the known error values, we can then obtain the value :

$$m_1 = \sqrt{0.0019 + 0.0036 + 0.0008} = \mp 0^s.073$$

As a result of this analysis, we can give : (1) the semi-definite longitude values obtained in 1947, 1948, 1949 by the astronomical field parties of the Army Map Service at the Ankara - Farm field center, using T4 Wild Universal Theodolites with impersonal micrometers, special radio and chronograph sets for meridian time observations; (2) the average latitude values determined at the same point and by the same apparatus and equipment, using the Horrebow-Talcott method.

These values with their inherent errors are as follows :

$$L_{s.\ def.} = 2^h11^m09^s.72 \pm 0^s.02 \text{ sec } \varphi \text{ east}$$
$$\varphi_d \text{ value} = 39°55'43''.18 \pm 0''.11 \quad \text{north}$$

Thus, the difference between the values obtained by the Wild T3 with prism astrolabe and the definite values, due to the method, apparatus, random, personal and systematical errors, is :

CORRECTIONS

Page	98,	line	9,	«geographical» (in place of «geophysical»)
»	98,	»	18,	«geographical» (in place of «geophysical»)
»	100,	»	26,	«ϕ» (in place of «φ»
»	100,	»	40,	Star pair No. «870-664» and «743-664» on bottom line of page No. 100 should be replaced on top of page No. 101
»	102,	»	24,	«. . . — tg φ tg Decl.» (in place of «. . . . — tg φ Decl.»)
»	108,	»	20,	«. . . — 2 cos φ cos Decl sin^2 t/2» (in place of «. . . — 2 cos φ cos Decl sin sin^2 t/2 »)
»	112,	»	27,	«Z» (in place of «z»)
»	112,	»	31,	«$Z' = Z + Ref_o + dr$» (in place of «$z' = z + Ref_o + dr$»)
»	112,	»	33,	«$Z'' = Z + Ref_o$ » (in place of «$z'' = z + Ref_o$ »)
»	112,	»	37,	«$Z = Z_o'' + dz$» (in place of «$z'' = z_o'' + dz$»)
»	113,	»	3 and 4,	«Z_o''» (in place of all «z_o''»)
»	113,	»	3,	«. . . + cos φ_o sin Z_o'' sin A_o dt — . . .» (in place of «. . . + cos φ_o sin z_o'' sin A_o . . . .»)
»	113,	»	8,	«. . . —sin Z_o'' (dz+dr) = o» (in place of «. . . —sin Z_o'' . . .»)
»	113,	»	16,	«. . . —sin Z_o'' (dz+dr) = o» (in place of «. . . — sin . . . .»)
»	116,	»	10,	«$m_{\Delta u} = m_\varphi /15 \cos \varphi_o$» (in place of «$m_{\Delta u} m_\varphi {}^{15} \cos \varphi_o$»)
»	117,	»	5,	«No. 116» (in place of «No. 21»)
»	119,	»	22,	«latitude» (in place of «zenith-distance»)
»	119,	»	27,	«chronometer» (in place of «hour»)
»	120,	»	2,	«c_o» (in place of «c»)
»	120,	»	4,	«c_o^2» (in place of «c^2»
»	121,	»	5,	«φ» (in place of «ϕ»)
»	127,	»	26,	«$m_s . \sqrt{\frac{i}{(\Delta T)^2}} = 0\overset{s}{.}06 \sqrt{\frac{2}{(10.1)^2}}$ » (in place of «$m_s . \sqrt{\frac{i}{(\Delta T)^2}} = 0\overset{s}{.}06 \sqrt{\frac{2}{(10.1)^2}}$ »)

Longitude; (Author-Semi-definite value) = $0^s.06 \pm (0^s.073 - 0^s.02)$ sec φ
Latitude; (Author-Definite value) = $+ 0''.04 \pm (0''.87 - 0''.11)$

Hence, as a result, it can be stated that this method is faster and furnishes an easier way of computation than the methods of John Ball an Th. Niethammer, provided that the approximate values for the author's method are obtained either from the existing topographical maps or determined by the secondary methods.

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